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MONOGRAPH

INNOVATIVE K-8 CURRICULUM FROM THE ARBOR SCHOOL OF ARTS & SCIENCES

ANCIENT GEOMETRY
INTERMEDIATES ENCOUNTER ERATOSTHENES

by Daniel Shaw

Supplements to this article:

Every other year, the 4th- and 5th-grade Arbor Intermediates survey the history of civilization. As this Inventions & Discoveries year unfolds, we study innovations in writing, language, science, architecture, and mathematics. From the development of the base-10 number system to proofs of the Pythagorean theorem, students engage with a variety of rich and ancient mathematical ideas. One of the most intriguing puzzles has to do with the work of a man named Eratosthenes. Born in modern-day Libya in the 3rd century BCE, this polymath invented the discipline of geography as we know it and contributed to many realms of academics. He rose to become chief librarian of the Great Library of Alexandria. Around 240 BCE Eratosthenes undertook a project to estimate the circumference of the earth. His figure was remarkably accurate: within 2% of the actual measurement. The key to Eratosthenes's work lies in one simple theorem of geometry: alternate interior angles are equal. Our Intermediate students slowly build toward a simple proof of this concept and use it to recreate Eratosthenes's argument that derived the circumference of the earth.

We lead Intermediate mathematicians through the following sequence of ideas:

Beginning angle work:
- What is an angle?
- How to use a protractor
- How to estimate angles using "friendly" angles like 90°, 180°, 270°, and 360°
- Parallel and perpendicular lines

Sum of angles in a triangle:
- Students work in pairs to investigate the properties of the angles of a triangle. After experimenting with many different types of triangles they become convinced that the sum of the angles of a triangle always equals 180°.
- They also begin to find unknown angles, using the facts that two angles on a straight line sum to 180°, angles around a point sum to 360°, and angles in a triangle sum to 180°.

Vertical angle theorem:
- When two lines intersect, they create four angles. Each two opposing angles are equal and are said to be vertical.
- Students work with many different examples to explore why this theorem is true.

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Alternate interior angle theorem:

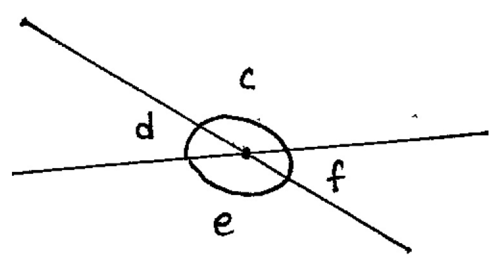
- When a transversal crosses two parallel lines, the angles on opposite sides of the transversal but between the parallel lines are equal.
- Again, students play with many examples in order to test this principle for themselves.

Throughout the unit, students add to their personal Math Toolkits to explain new terms, tools, and concepts.

When we introduce our students to ancient math, one of our hopes is to instill awe at the human ingenuity that produced some of the calculations and conclusions that are central to mathematics today. The fact that a librarian living 2400 years ago estimated the circumference of the earth with nothing but a pole, a protractor, and a shadow provides plenty of meat for astonishment. In order to understand Eratosthenes's methods, students have to engage with the concepts that supported his conclusion. Eratosthenes's key realization pertains to parallel lines: if two parallel lines are crossed by another, they create sets of equal angles. More formally, two sets of alternate interior angles are equal. Students require some background knowledge in order to discover this property themselves, but the concepts in play are accessible to 9-, 10-, and 11-year-olds.

Initially, we work with students to build protractor skills. The Intermediates practice using a protractor to measure all sorts of angles—those between 0° and 180° and those greater than 180°. The fact that protractors are constructed to allow measurement of angles beginning at either side can be an obstacle at first. When students ask, "Which number do we use?" we ask them to relate the angle to 90°. If the angle is greater than right, they need to use the larger number. We then spend an entire class period estimating angles. Students begin to call certain angles—90°, 180°, 270°, and 360°—"friendly" because they are particularly helpful in estimation. They learn how to describe the relationship between two lines, whether perpendicular, parallel, or simply intersecting. Students investigate acute and obtuse angles and come up with names for 180° and 360° angles. They record all of these findings in their Math Toolkits to scaffold future learning. Once all students have a common language and skills to work with angles and lines, we begin to investigate the properties necessary to understand Eratosthenes's work.

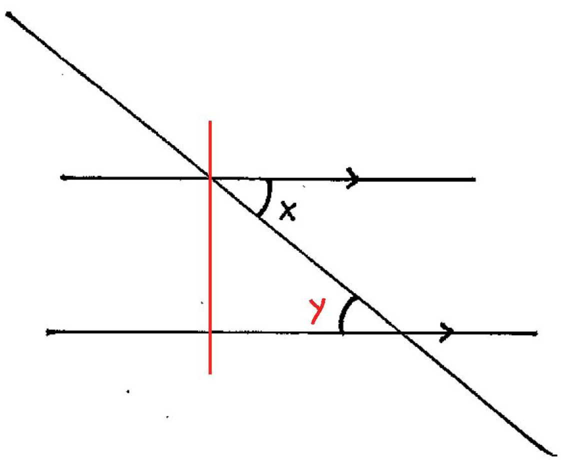
Eratosthenes used two related geometric theorems to obtain an estimate for the circumference of the earth. First, he satisfied himself that vertically opposite angles are equivalent. (A Greek philosopher named Thales of Miletus had worked out a proof of this about 300 years before.) For example, in the figure below, angles c and e are equal; so are angles d and f.



Our Intermediates work in pairs through a series of increasingly complicated problems to come to the same conclusion. They begin with two angles along a line. We give them one angle and ask them to calculate the other. Although all our students have learned that two angles along a straight line sum to 180°, it can be tough for them to leap from that fact to the subtraction problem that will yield the measurement of the missing angle. We work with numerous examples and many different angles in order to build confidence in applying subtraction in this way. Eventually, students explore the four angles created by two intersecting lines. Given the size of one angle out of the four, students can build sums to 180° in order to calculate the measurements of the other three. After working through multiple variations on this task, students feel certain that the opposing angles formed by two crossing lines are always equal. It helps to ask them to explain their thinking along the way, making sure they know why they completed each step of the process.

See the appended Angle Worksheets beginning on p. 11.

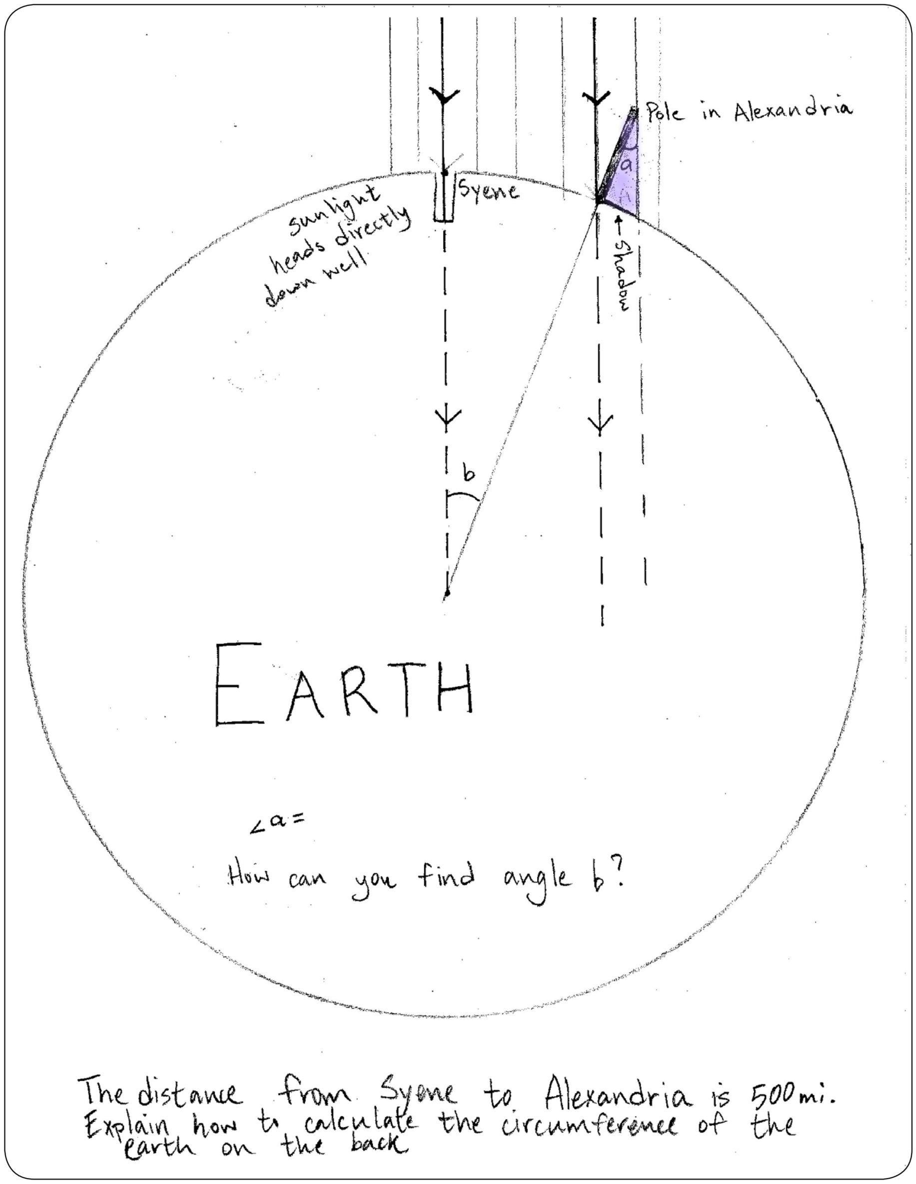
The second theorem that Eratosthenes needed relies on the vertical angle theorem and on the fact that the sum of the three angles in a triangle is 180°, a fact Intermediates have already encountered and tested in previous lessons. In order to learn that alternate interior angles are equal, student begin with a transversal crossing two parallel lines. We show them how to drop a perpendicular line through the intersection of the transversal, creating a triangle:



We have students measure angle x and then calculate as many other angles as they can, allowing them to arrive in their own time at the realization that x and y are equal. Again, our students work with multiple examples of this property and summarize what they have discovered at the end. Group discussion is usually the best way to lead all the students to the idea that if x + b = 90 and y + b = 90, x and y must be equal. We have noticed that students tend to use the triangle formed by the perpendicular line as a crutch and attempt to draw it themselves in every problem, thinking it is a necessary part of the puzzle. But once they accept that x and y—alternate interior angles—are always equal, they can see that the theorem still holds whether the line is there or not. Armed with these tools, students are ready to emulate the work of Eratosthenes.

The idea that a mathematical proof can only be made by truly exhaustive testing is new to them. Students this age are, if anything, *too* ready to assume a proof from a handful of examples. When they reach the Senior level we ask them to exercise greater caution in concluding that a pattern will always continue.

After a read-aloud of the first part of Kathryn Lasky's *The Librarian Who Measured the Earth*, each student receives a drawing that represents a view of the globe that Eratosthenes imagined—a section of the earth from Syene to Alexandria, with rays of sunlight running parallel into a well in Syene and casting a shadow of a pole at Alexandria.



Lasky's story tells us that Eratosthenes might have visualized the earth as being segmented like a citrus fruit. He realized that if he could measure the size of one section of the earth and find out how many of that section would fit around the whole planet, he could calculate the circumference. With little scaffolding, the Intermediates can find their way to the calculations that Eratosthenes undertook. The toughest step is seeing how Eratosthenes figured out the number of sections he would need. Helping students is often as simple as reminding them that they know a circle is 360°; they can then usually turn the question into a division problem. They can use either a calculator or long division to obtain the number of sections that would circle the globe.

In the end, students marvel that Eratosthenes's estimate differs from our own modern estimates by only 200 miles—this without any computers or technology to speak of. (Eratosthenes used surveyors trained to walk with very precise strides to measure the 500 miles from Alexandria to Syene!) The study of Eratosthenes's achievement also makes a great segue into our study of Medieval and Renaissance times. Sixteen hundred years later, European scholars still revered and relied upon the work of the ancient Greek philosophers. As kingdoms vied for control of trade routes and newly encountered lands in the Age of Discovery, Eratosthenes and his contemporaries were guiding lights in the sciences of navigation and geography. Why hadn't human understanding advanced further in the intervening centuries?

Angles Pre-Assessment

Name ______________

Show your understanding of the following math sentences by drawing labeled diagrams:

$AB \perp CD$

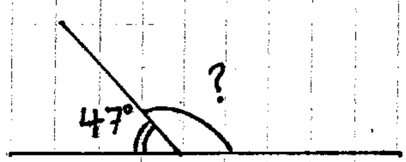
$EF \parallel GH$

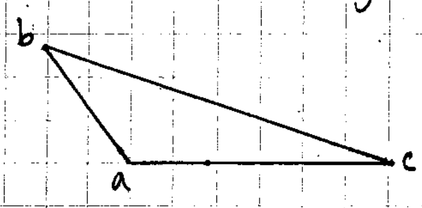
$\angle ABC$ is acute.

$\angle DEF$ is obtuse.

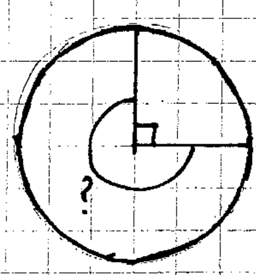
Draw an isosceles right triangle named $\triangle XYZ$

Calculate the measurement of the unknown angles.

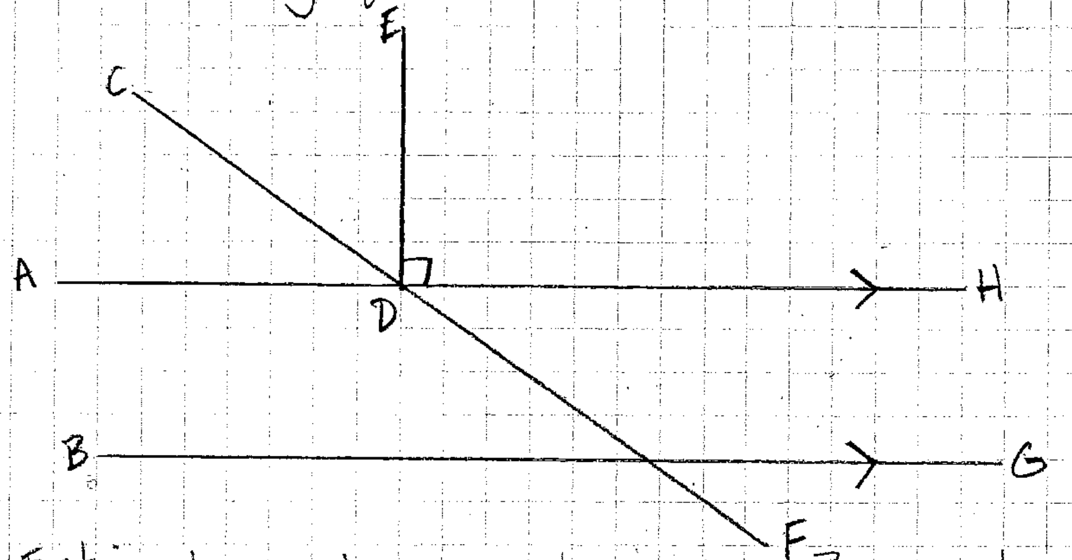




$\angle a = 128°$
$\angle b = 35°$
$\angle c = ?$



Reference the following diagram to answer the following questions:



1) Estimate and properly name 3 angles:

2) Use a protractor to measure and properly name 3 different angles:

3) Describe this diagram with as many math sentences as you can:

Define the following math vocabulary words:

perpendicular:

parallel:

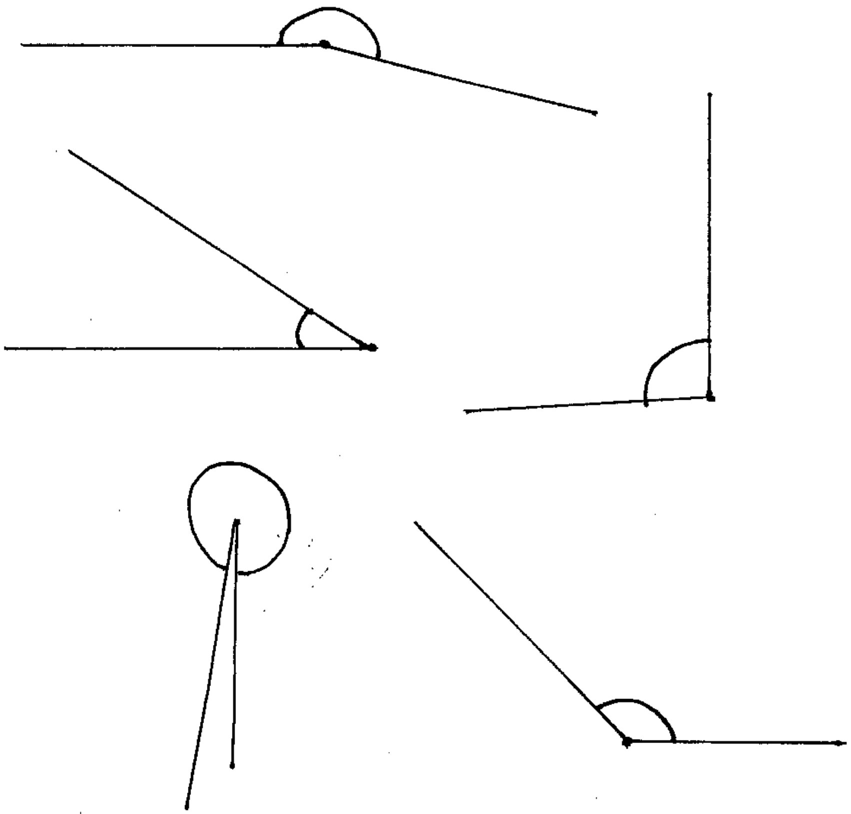
acute:

obtuse:

If you still have time, create an interesting diagram to label and describe with math sentences.

Estimate these angles.

Name: ____________

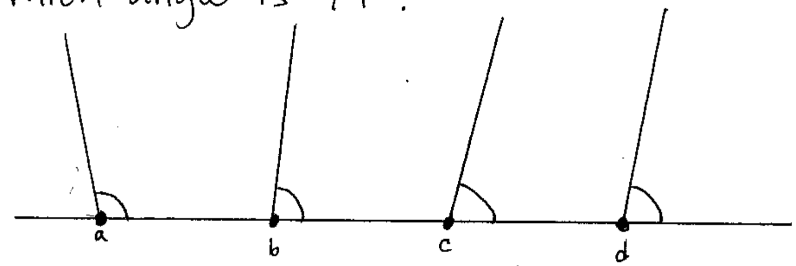


On the back, draw approximations of these angles:

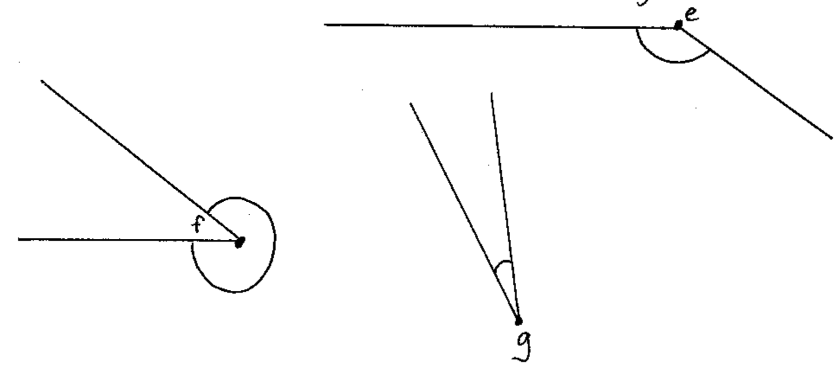
95°
30°
270°
125°

Name ____________

Which angle is 79°?



Use a protractor to measure these angles:



Draw these angles:

h.) 63°

i.) 335°

Name ________________

After you have practiced using a protractor on angles on a piece of paper, it's time to measure some angles around the room. Find 10 different angles to measure around the classroom. Record what you measured and the size of the angle below:

1 ________________________________

2 ________________________________

3 ________________________________

4 ________________________________

5 ________________________________

6 ________________________________

7 ________________________________

8 ________________________________

9 ________________________________

10________________________________

Name ________________

After you have practiced using a protractor on angles on a piece of paper, it's time to measure some angles around the room. Find 10 different angles to measure around the classroom. Record what you measured and the size of the angle below:

1 ________________________________

2 ________________________________

3 ________________________________

4 ________________________________

5 ________________________________

6 ________________________________

7 ________________________________

8 ________________________________

9 ________________________________

10________________________________

Name ________________

Protractor Math Toolkit Entry

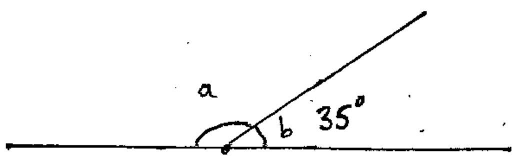
Now that you've learned how to use a protractor we want to make sure you don't forget! One of the best ways to do that is to write it down. Create a beautiful and clearly written math toolkit entry that includes the following information:

1) A drawing of a protractor that has marks for every 5 degrees. Either trace a protractor or use a compass and ruler to make your drawing. Make sure to start counting at each side just like a normal protractor.
2) Tell your reader how to hold the protractor and where to orient it with respect to the angle you want to measure.
3) Explain how to read the protractor and how to write degrees.
4) An explanation of how to draw an accurate angle using the protractor.

Name__________

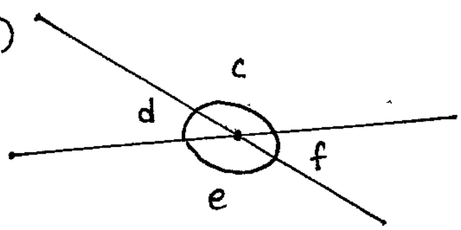
1) 

Measurement of a straight angle: ______

2) 

$\angle a + \angle b =$ ______

$\angle b = 35°$, therefore $\angle a =$ ______

3) 

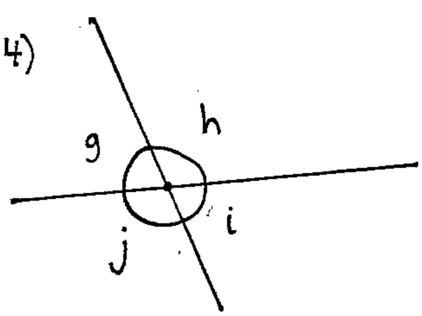
$\angle c + \angle f =$ ______ ⇓ SHOW YOUR THINKING ⇓

$\angle c + \angle d =$ ______

→ measure $\angle c$: ______

* Therefore, calculate $\angle f$ ______

$\angle d$ ______

4) 

$\angle j + \angle i =$ ______

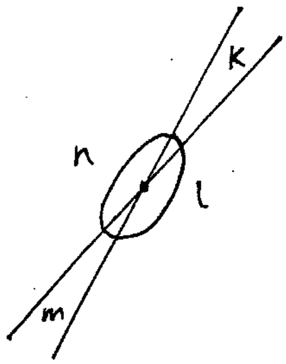
$\angle j + \angle g =$ ______

→ measure $\angle j$: ______

* calculate $\angle i$: ______

$\angle g$: ______

5)



$\angle k + \angle l =$ ______

$\angle k + \angle n =$ ______

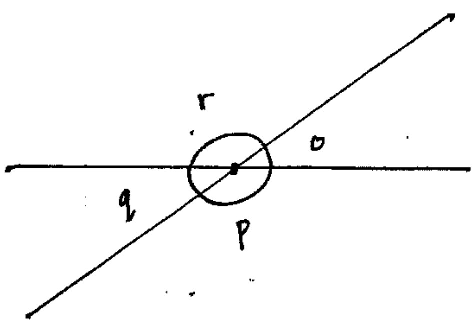
→ measure $\angle k$ ______

* calculate $\angle n$ ______

$\angle l$ ______

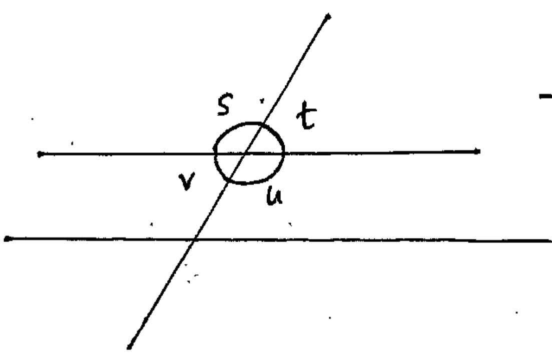
Without measuring, how could you figure out the size of $\angle m$? Explain.

6)



Given $\angle o$ is 37°, what do you know about the other angles shown? Please be specific, and explain your thinking.

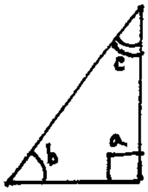
7)



→ Measure one angle and calculate as many other angles as you can. Explain how you determined your answers.

$\angle$ ___ measured	other $\angle$s
$\angle$ ___ = ___°	

Name ______________

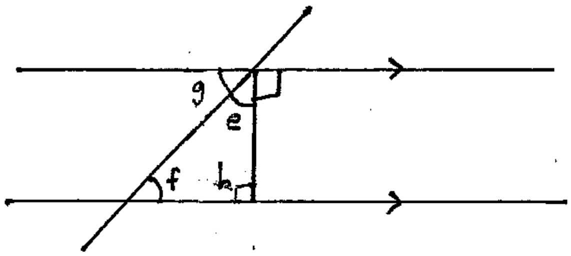
8.) 

$\angle a + \angle b + \angle c =$ ______

$\angle a =$ ______

Therefore,

calculate: $\angle b + \angle c =$ ______

9.) 

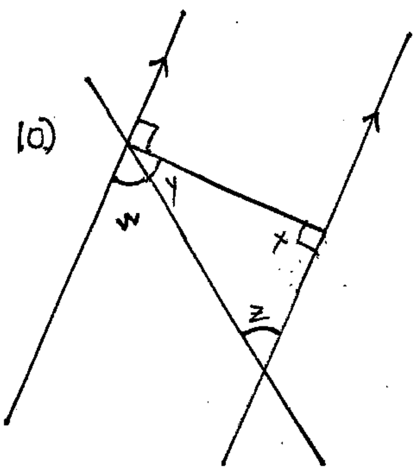
$\angle e + \angle f =$ ______

$\angle e + \angle g =$ ______

→ measure $\angle e$ ______

* calculate $\angle g$ ______

$\angle f$ ______

10.) 

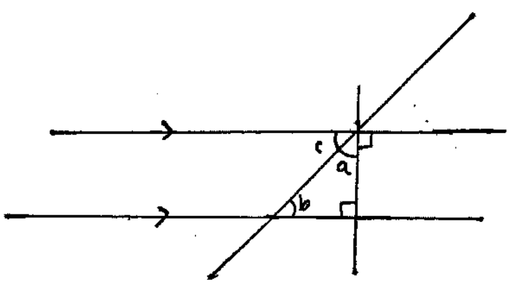
$\angle y + \angle z =$ ______

$\angle y + \angle w =$ ______

→ measure $\angle y$ ______

* calculate $\angle z$ ______

$\angle w$ ______

11.) 

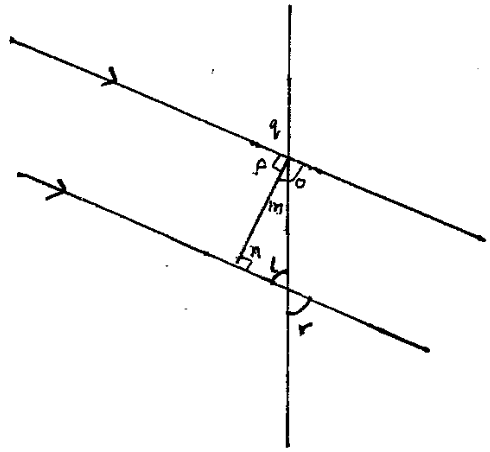
Given $\angle a = 45°$,

What is $\angle c$? ______

What is $\angle b$? ______

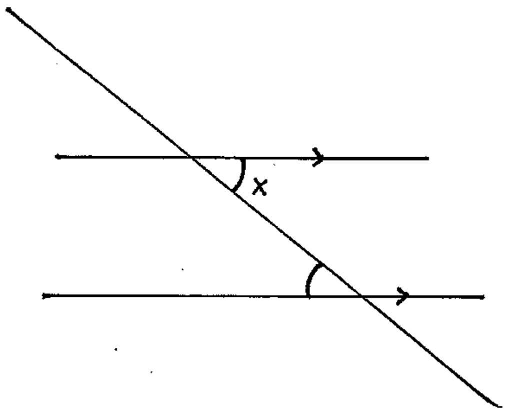
Explain your thinking:

12.)



Given ∠l is 66°, how many other angles can you determine without using a protractor? List/show your calculations:

13.)

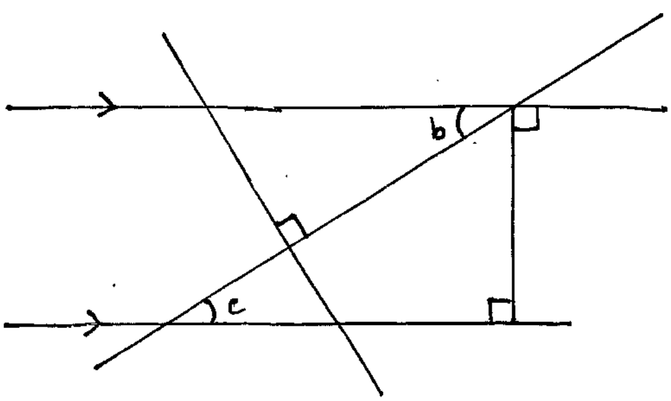


Measure ∠x: ______

Use that information to find as many other angles as you can, using logic and calculation.

Show your method.

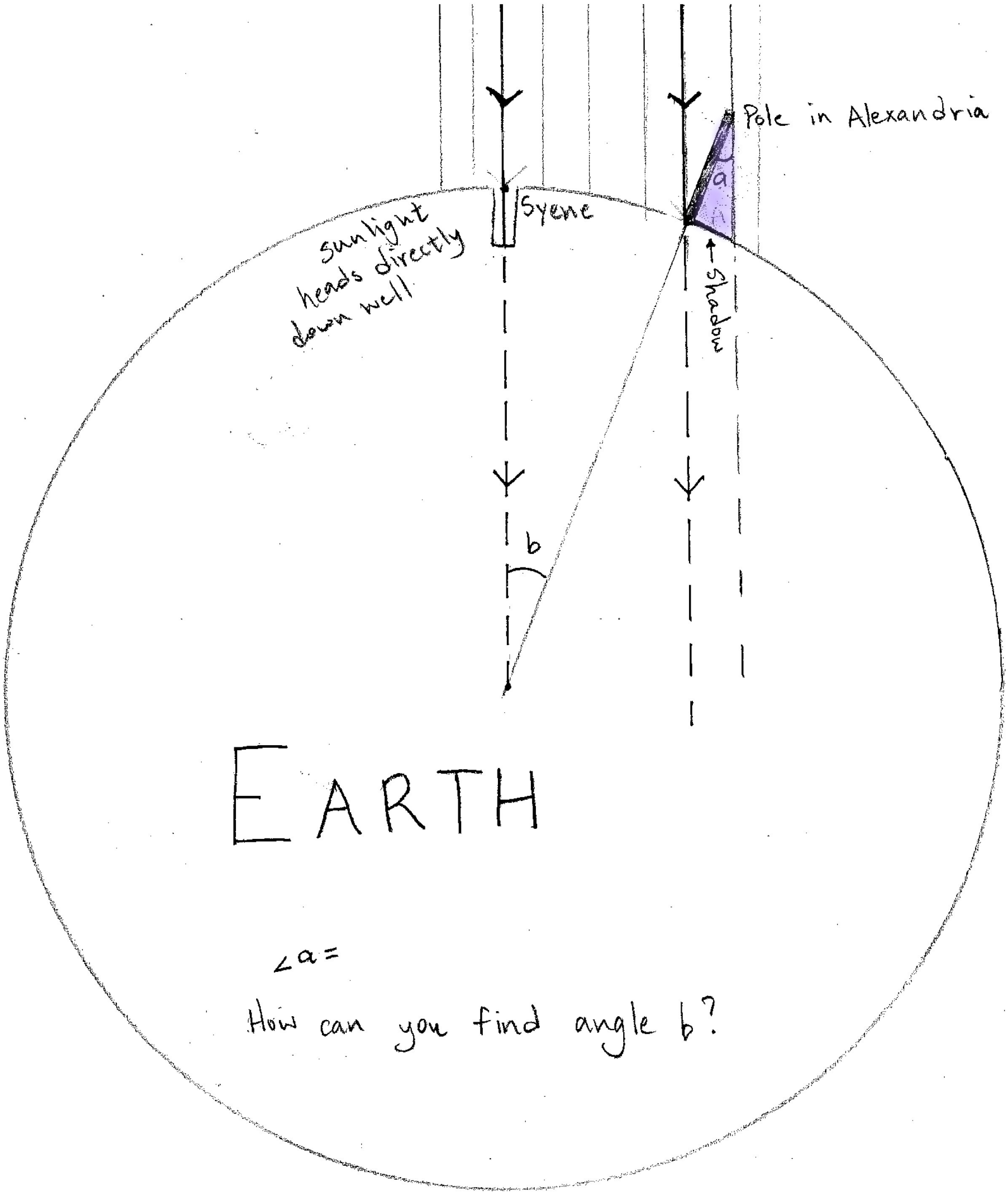
14.)



∠b is 32°

What is ∠c? ______

What are the other angles you can figure out using that information?



$\angle a =$

How can you find angle b?

The distance from Syene to Alexandria is 500mi.
Explain how to calculate the circumference of the earth on the back

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Cambium: (n) the cellular growth tissue of trees and other woody plants, from medieval Latin "change; exchange."

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Masthead by Jake Grant, after an 1890 botanical illustration.

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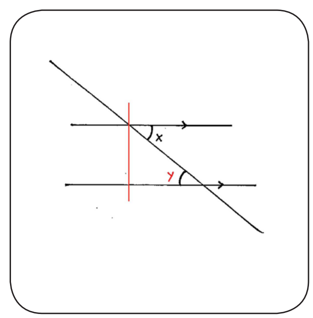

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