Cambium

VOL. 1, NUMBER 1

INNOVATIVE CURRICULUM FROM THE ARBOR SCHOOL OF ARTS & SCIENCES

In this issue

ARBOR SCHOOL
OF ARTS & SCIENCES

BASE TEN AND BEYOND

The number system we begin to learn as toddlers, counting our ten fat fingers, has roots 5,000 years old. The carvers of Egyptian hieroglyphs from 2900 BCE counted their goats in powers of ten. The Chinese, too, developed positional notation, and by 1300 BCE recorded dates as "five hundred plus four decades plus seven of days" (547). In India great mathematical leaps were taken, and by 900 CE scholars had constructed our modern number system, complete with decimals, negative integers, and zero. These Hindu-Arabic numerals were transmitted to Persia and the Arab world, and in 1202 Leonardo Fibonacci, son of the director of a trading post in North Africa, finally introduced them in Europe, where their simplicity, efficiency and power ousted the Roman numeral system.

But not all cultures have used ten as the base for their number systems. The Mayans made their astounding astronomical calculations in base twenty. The Babylonians used a base sixty notation. Base twelve survives in some Nigerian cultures. Computers work in base two. In order for students to become capable manipulators of our base ten system, they must understand how and why our positional notation works, and grasp the meaning of "place value."

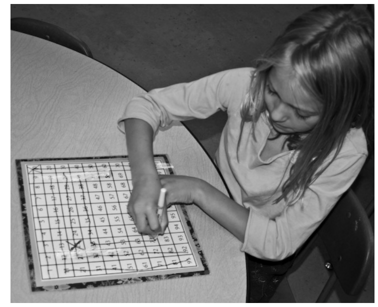
Teaching about the positions of units, tens, and hundreds begins in the earliest days of elementary school. At Arbor, we are fortunate to have a Primary (kindergarten and first-grade) teacher who is also a passionate mathematician, Lori Pressman. Lori's lessons introduce the hundreds' chart as a trusty tool for children as they build numeracy, and as the backdrop for a bushel of engaging number games. At the Junior (second- and third-grade) level, Peter ffitch and Janet Reynoldson lead budding geologist-mathematicians to explore the possibilities of negative integers and absolute value, digging into the soil and evaluating the records from the drilling of two wells on school property.

By the time our students have reached the fifth grade, their command of the basic operations in mathematics has prepared them to tinker under the hood of our numeral system. Nothing deepens an understanding of place value in base ten more quickly than attempting calculations in a different base system – one of the student's own devising – as Linus Rollman asks them to do. And as the sixth graders make forays into algebra, Linus introduces them to exponents – powers of ten – as tools for managing the very largest and smallest numbers, even when those numbers are unknowns. A Renaissance man himself, Linus offers his Senior students a chance to apply their knowledge of exponents to an unlikely puzzle: one of poetry.

DIRECTIONS TO NUMERACY
PRIMARIES LEARN TO NAVIGATE THE HUNDREDS' CHART

by Lori Pressman

Materials:
- Small hundreds' chart for each child, laminated
- 11″ x 11″ hundreds' chart for each child
- Large hundreds' chart for the class
- Hundreds' chart with pockets for removable pieces (see Resources)
- Highlighters
- Overhead markers
- Unifix cubes
- Cards labeled 1-100
- Overhead pens
- Glass drops, a.k.a. cars



During our annual staff retreat, the Primary team talked about written language acquisition and how readily the kindergarteners take to guess-and-go writing. In their knowledge of phonemes, the alphabet chart on the wall, and the sound/letter charts at their worktables, they seem to have all the tools they need to get started. We started to wonder if there were tools of equivalent importance for numeracy. What resources were readily available in their environment to assist our students in mathematical problems? The kids rarely sought the number line boldly written on the floor or the hundreds' chart in our calendar corner. But like learning the alphabet and its corresponding phonemes, learning how to navigate a hundreds' chart and number line would help them build a solid foundation for number recognition and concepts such as patterns, computation, place value, and logical deduction. How could we introduce them to the beauty, logic, and value of number lines and hundreds' charts?

The hundreds' chart in particular intrigued me. But before I could start building a set of lessons, I needed to debate the merits of a 0-99 chart versus a 1-100 chart. The 0-99 chart has all ten digits neatly recorded on the top line; the subsequent rows are filled with double digit numbers all beginning with the same digit in the ten's place: 40, 41, 42, 43, 44, 45, 46, 47, 48, 49. It is easy to recognize the developing patterns. Children would predict, "This whole row is going to be in the sixties." The challenge with this chart occurs when it is completely filled: all 100 spaces are occupied, but the largest number is only 99. It also proved tricky to use a 0-99 chart to teach place value. Although I did use 0-99 charts last year, I've determined that the 1-100 chart would be a more helpful and logical teaching tool. Ultimately, what convinced me is that each space in a 1-100 chart actually represents a number and the rows are complete sets of ten: 74 is represented by seven rows of ten plus four units. Using 1-100 charts in conjunction with base ten blocks could be even more potent, especially if the 1-100 charts were the same dimensions as the base ten blocks.

The following lessons were created with the 1-100 chart in mind. They will acquaint young mathematicians with this fundamental math tool, providing a solid base for exploring patterns such as counting by twos, threes, fives, nines, elevens, etc. Children can quickly appreciate the friendliness of +10 or +20 or +30. Using a 1-100 chart for games like Guess-Digit-Place (see Sidebar) helps kids develop deductive reasoning skills by letting them visualize the diminishing number of choices. The hundreds' charts starred in our math lessons for several weeks last winter, and over time the kids began to rely on the new tools. Whether they were pointing to numbers as we counted by 20's to 100 or adding nine by moving ↓ ←, the hundreds' chart that was once a wall decoration beside the calendar was now an interactive resource.

In Guess-Digit-Place, one player thinks of a number and draws a table of three columns. He records the guesses in the first column, the number of correct digits in the second, and the number of correct places in the third. The guesser crosses off the numbers eliminated on a 1-100 chart. If the number is 53 and the guess is 37, there is one correct digit (3), but no correct place (the 3 is in the wrong position). To check which digit is right, the guesser might propose 72 next: no correct digit or place. All numbers containing 5 or 7 are now eliminated, as are numbers with 3 in the tens' column. Play continues until the guesser has found the mystery number.

FORECAST
Preassessment:

Working individually with students, ask them to count a set of objects that is greater than thirty. Take note of strategies they use to count. Do they count by twos? Do they group in sets of ten? Ask the students to fill out a blank hundreds' chart. Take notes on how they approach the task. Do they write the numbers

consecutively? Are they writing the numbers in columns—10, 20, 30? Are they recognizing patterns and writing all the fours in the one column? What happens when an error is made? How do they discover it and how do they correct it?

Lesson 1: Reading the Map

Give each student a hundreds' chart and, as a class, point to and read aloud each of the numbers. Ask a few students to share their favorite number; the others should hunt on their charts and point to the number. As everyone looks for these favorite numbers, ask, "What is your secret to finding 45?" This is a great time to emphasize the terms "tens' column" and "ones' column," and to introduce the terms "number" and "digit." It may help to say that digits are similar to letters and numbers are similar to words. Some words have just one letter, such as *a* and *I*; some numbers are single-digit numbers. To give the students practice with these terms, ask them to locate a single-digit number or a double-digit number. Make sure to ask for examples. Then ask them to point to a number with a four in the ones' column or a seven in the tens' column. If pointing to numbers is getting too tedious, the students can use different colored pens to cross out or circle particular numbers.

Lesson 2: BINGO

After distributing hundreds' charts to all the kids, start by asking them to find their weight on the hundreds' chart. You may want to bring in a scale they can use to weigh themselves. Ask kids to share their numbers and then find the weights of their friends. After finding a few numbers, introduce BINGO to the kids. Because the hundreds' chart is so large, the goal is simply to get five numbers in a row, vertically or horizontally. Dispense Unifix cubes and begin by selecting one of the 1-100 cards from a basket. Read the number aloud and then mark it off on the large hundreds' chart. Start slowly and make sure students are able to locate and cover the numbers on their own charts with Unifix cubes. Asking neighbors to check in with one another is one way to manage accuracy. Every once in a while, ask, "Can anyone explain how he/she found 37? Did anyone have a different strategy for finding 37?" Also, when a BINGO is nearing, it is fun to point it out on the large group chart: 6, 16, ___, 36, 46, 56,___,___. Ask the children what numbers are missing and how they know. Make sure to reinforce the terms "tens' column" and "ones' column" when pointing out the numbers. Continue playing until you have five in a row. You're all winners!

Lesson 3: Too Low-Too High

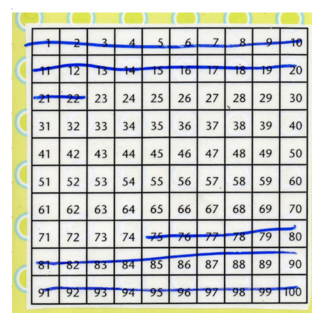
We play Too Low-Too High all year on our ever-expanding hundreds' chart. For instance, on the 26th day of school, we searched for a mystery number between 1 and 26. Today the kids will be searching for mystery numbers between 1 and 100. Using large hundreds' charts that are copied onto 11" x 17" paper, they will narrow their search using logical deduction. Crossing the numbers out after each guess will help the children visually restrict their choices. Start by creating a Too Low-Too High T-table on the chalkboard and secretly select a mystery number, such as 58. Ask the children to guess it. In this case, 22 was the first number guessed. Record 22 in the "Too Low" column and cross out 22 and all the numbers less than 22. The next guess is 75; it is recorded in the "Too High" column and 75 and all numbers greater than 75 are crossed out. Now the search is narrowed between 23-74. Play a few rounds of Too Low-Too High. Eventually the children will be able to choose the secret numbers and record guesses while their guessing partners cross off the eliminated numbers.

Lesson 4: Filling in the Holes

Scavenger hunts are always fun, and the hunt for numbers is no exception. As the children find the number cards, they put them in order on the blank hundreds' chart. Having five or six cards already in place helps to establish a few landmarks. There are

Field Notes:
I asked only the first graders and a handful of capable kindergarteners to work with the blank hundreds' chart at this point. The assignment is overwhelming for young children who haven't developed strong number sense.

When counting teen numbers, I often refer to 11, 12, 13, 14... as tensy-one, tensy-two, tensy-three, tensy-four, etc. We talk about how tricky it is to hear the difference between fourteen and forty and to see the difference between 14 and 41. There seems to be a sense of relief for kids knowing that they're not alone if they get tripped up on these.



Behind the Scenes, Lesson 4: Write the numbers 1-100 on brightly colored cards or slips of paper and hide them throughout the classroom.

many variations on this warm-up activity. For Valentine's Day, we began class with a hunt for 100 numbered paper hearts. Another day, a mischievous school troll had mixed up the numbers on the hundreds' chart, and the kiddos had to do some damage control. Looking for the errors on the chart helps to solidify the students' understanding of its inherent patterns.

As the children come to the chart, ask them how they know where to put the number. Do they count on from a landmark number? Do they use their knowledge of patterns to navigate vertically? Are there any misplaced cards? How and when are they discovered?

Field Notes:
This becomes a faster paced game and we find we can give 3-5 directional symbols at a time. After a few rounds with the teacher giving driving instructions, the kids are ready to lead each other.

Lesson 5: The Road Map

Today we are going on a math road trip and the kids need two items for the trip: a map and a car. A laminated hundreds' chart is the map; each child chooses a "vehicle" from a collection of colorful glass drops. Each kid places her car on a particular number and then, following a series of instructions, the cars move around the hundreds' chart. Write the starting number on the board and then write a directional arrow. If you begin on 43, give the instruction to ↑: now the cars should be on 33. Instruct the children to move → and the cars move to 34. Begin slowly and check after each move. The children not only become acquainted with the chart, but also get practice reading numbers.

27 ↓ ↓ ↓ → Where are you? 58!

Behind the Scenes, Lesson 6: Enlarge the "Excuse me, could I please have directions to…?" worksheet and make enough copies for each child.

Lesson 6: Writing Directions

Today the children have a chance to write their own directions to different numbers. Using their hundreds' chart maps and glass drop cars, the students write the directional arrows to travel from one number to another. Demonstrating first on the board, ask the kids how to get from 24 to 4. They should write ↑ ↑ in the boxes between the two numbers. Once everyone understands the assignment, pass out the worksheets so they can practice. Having the map and car at their fingertips helps the kids determine and confirm the directions they have written.

Note which children are able to write directions without the concrete model.

Behind the Scenes, Lesson 7: Enlarge the "What numbers are missing?" worksheet and make enough copies for each child.

Lesson 7: Missing Numbers

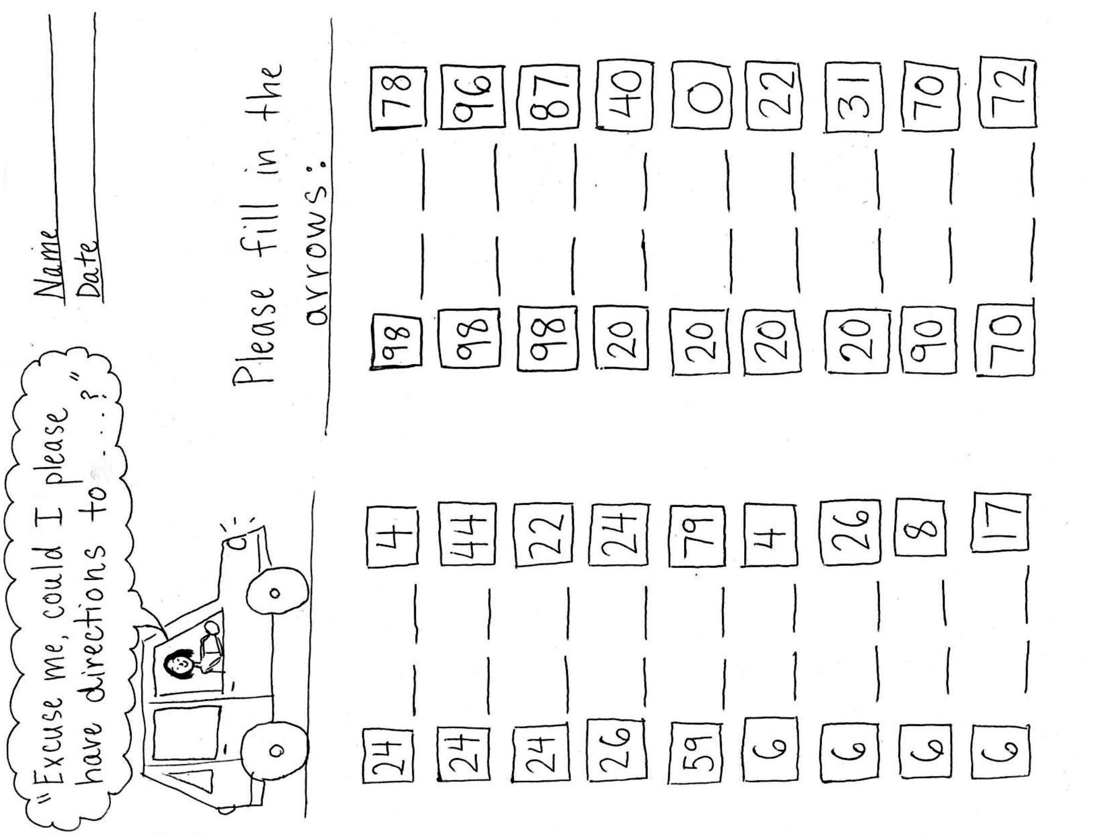
Cover some of the numbers on your large hundreds' chart with sticky notes, or erase some on a chart written in overhead marker. Ask the class to fill in the missing numbers on this partially complete hundreds' chart. You may wish to prepare a similar worksheet for extra practice. When they are comfortable filling in missing numbers, distribute the "What numbers are missing?" worksheet, which shows puzzle-like pieces from a hundreds' chart.

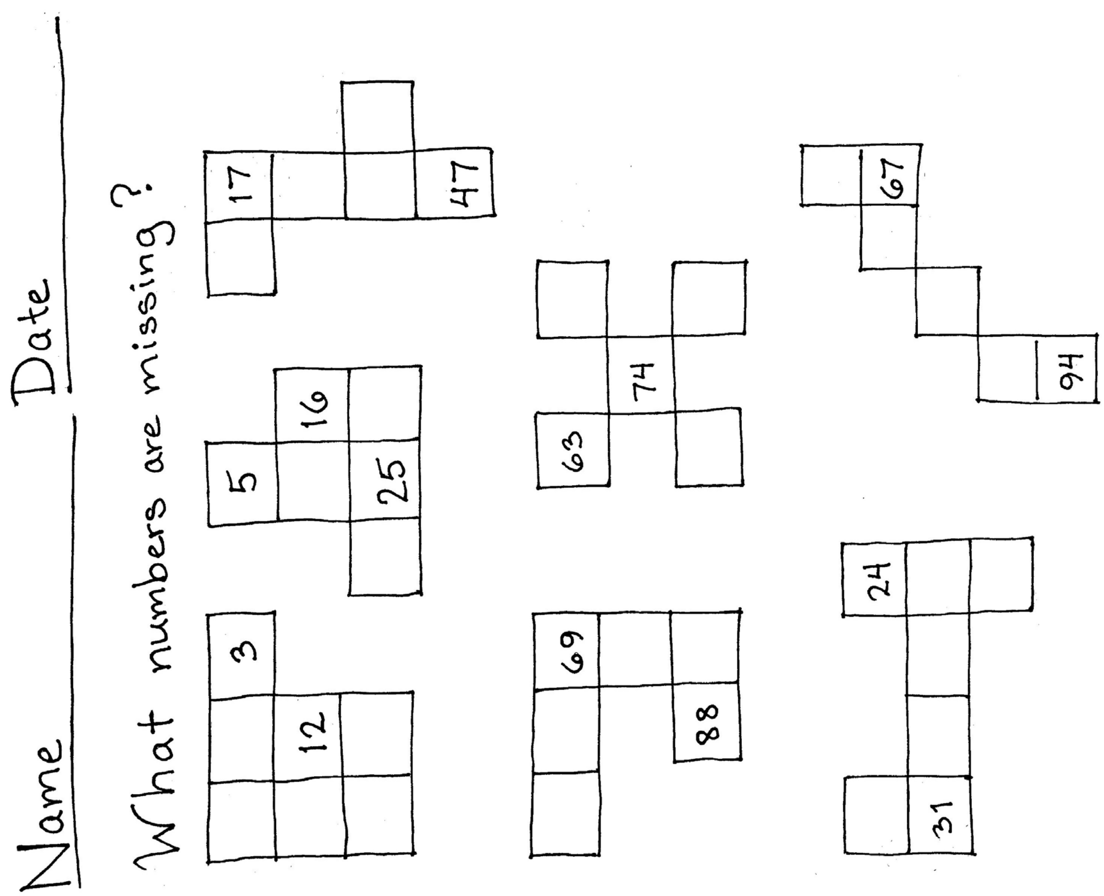
Ask the children how they determine which numbers are missing. Are they using the terms "ones' place" and "tens' place"?

RESOURCES

Burns, Marilyn. *About Teaching Mathematics: A K-8 Resource*. Math Solutions Publications, 1992.
Marilyn Burns is one of our favorite sources for math teaching ideas and philosophy. Contains 0-99 charts, but not 1-100 charts.

Hundreds' wall charts with pockets, base ten blocks, and other manipulatives are available from Creative Publications (www.creativepublications.com).





Field Notes:
Knowing that I was going to embrace the hundreds' chart in my math curriculum, I wanted to make sure that the students would feel equally eager to experiment with these tools. I took time to make these hundreds' charts attractive—pretty paper and strong laminate. These durable little tools withstood many lessons and the kids had a great time using Vis-à-vis overhead pens on them. Although each chart took a little extra time to make, it was worth it.

I also like to incorporate math-related picture books during our read-alouds. A favorite is *Anno's Magic Seeds*, by Mitsumasa Anno.

Mathematicians pictured: Beatrice Pickett and Solomon Olshin with Lori Pressman

"Excuse me..." worksheet by Fran Hossfeld

DIGGING DEEP

JUNIORS USE A WELL LOG TO EXPLORE ABSOLUTE VALUE

by Peter ffitch

Materials:

- wood scraps
- wood glue
- electric drill
- well log (see Resources)
- art supplies

Under the thematic umbrella of Continuity and Change, we began the year with our Juniors (two blended second- and third-grade classes) looking at the natural processes that shape the earth, with a particular focus on the Northwest, Portland, and ultimately our school campus. In order to bring the study to life for the children, we gave them the opportunity to act as geologists on our school grounds. The children chose to dig in four locations, hoping each would yield something that would teach us about how the land under our feet was created. They dug with energy and returned to their sites as often as possible during a two-week period, but their shovels brought up only clay instead of the hoped-for rocks. Although this fact in itself led us to an interesting investigation into the source of these deposits, we still hoped to find a way to give the children a more complete picture of the geologic forces at work in this place. We found our tool in the school's well logs. These documents include detailed records of what the drills brought up as the two wells on campus were dug. As interesting as the list of varieties of clay, sand, shale, gravel, and basalt were to us, we also saw enticing raw math material in the way each layer the drill encountered was recorded. The report labels the surface as 0 feet and then proceeds to describe the composition and depth of each layer: "Soil, brown sandy, 0 – 3 feet." "Clay, brown, 3 – 20 feet." This format continues for 16 layers and to a depth of 307 feet.

This data presented a number of mathematical entry points. One possibility would be to create a vertical number line with 0 feet (surface level) as the mid-point. With this model children could be introduced to negative numbers. It would provide an opportunity to give the children a sense of the depth of the layers below them by relating them to the height of measured structures on the surface, such as a school building or a large tree. We chose instead to use the well logs to get at the meaning of "difference" in a new way.

FORECAST

Day 1:

Talk about the limitations of exploring the ground beneath our feet with a shovel. How deep do you think you could dig with just these simple tools? Explain that when people need to dig a really deep hole, to find clean water for a well, they must often dig hundreds of feet below the surface. To do this, they use machines with big drill bits. Demonstrate drilling with an electric hand-drill into the block of wood: lead the children to notice the corkscrews of material the drill forces out. If you were drilling into the ground, you could evaluate the different types of soil the drill brought up as it went deeper and deeper. Introduce the well logs.

Day 2:

Look more carefully at the data in the well log. Pair the children and give them problems such as the following: "If there is a layer of brown clay beginning at 56 feet below the surface and extending to 78 feet below the surface, how many feet of brown clay are there?" Framing the problem in this way challenges children's understanding of what subtraction is really all about.

Some kids may think the problem must be impossible. Others will find a negative integer solution and puzzle over how to make sense of that as a measurement of a physical distance. Do not penalize these reasonable conclusions, but do note how each child engages this new idea. How do those who "get it" explain to others?

Behind the Scenes, Day 1: If you can procure scraps of different woods (various colors will be most effective), glue them up into a block of four or five layers to simulate the strata a well-drill might encounter. A lumberyard will have scraps, or a parent with a woodworking hobby should be able to provide some interesting materials. But any block of wood will let you demonstrate the way the drill removes material from the hole. You might try a similar exercise by having the children layer colored clay, then take a core sample by pressing a plastic or metal tube into the clay.

Behind the Scenes, Day 2: Prepare a sheet of math problems based on the information in your well log: *How many different layers can you count? What is the description of the thickest/thinnest layer? How thick or thin are those layers? How many kinds of clay are listed? How many feet of basalt did the drillers find?* Each student will need a copy.

Their uncertainty provides an opportunity to explore the concept of absolute value. A vertical number line may be useful to help children see the value of a number as indicated by its distance from zero (in the case of the well log, zero is the surface). Remind them of other familiar concrete models, such as a thermometer that shows temperatures in degrees above or below zero. You might create a more provocative example by planting seeds below the surface in a transparent container. If the surface is zero, how do we use numbers to describe the new growth pushing up and the roots pushing down? What if the seed itself is zero? Children can count degrees on a thermometer or inches of growth in either direction from a seed and see those numbers as having "positive" or measurable values.

Using your concrete examples and the vertical number line, help the children see that the well log problem could be restated in a more familiar way, such as, "How many tens and ones do we need to add to 5 tens and 6 ones to make the sum 7 tens and 8 ones?," or, "What is 7 tens and 8 ones minus 5 tens and 6 ones?" Once these differences had been computed and recorded for each of the 16 layers of organic material, our children began to get a picture of the relative thicknesses of the layers and could approach the worksheet questions with greater confidence.

It is worth noting that by asking the children to use feet and inches as the units of measure, you are asking them to use a number system that isn't base ten. You may wish to take this opportunity to introduce the concept of different bases.

Days 3-4:

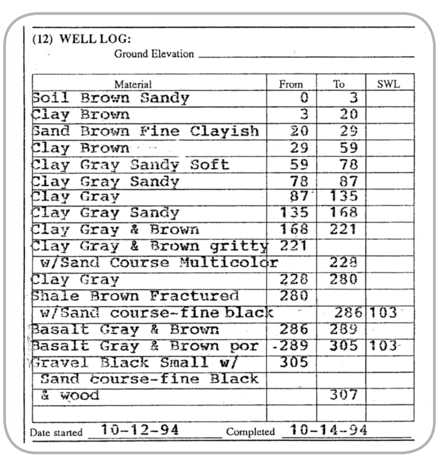
To make the information in the well log more concrete, announce an art project to make a visual model of what you have discovered. Let the students participate in finding an appropriate scale that will allow you to display your model on the wall of your classroom. (Our children settled on a scale of 1ft. = 1/2 in.) Assign pairs of students to work on each layer.

The children now translate the actual measurement of their assigned layer into a display-friendly measurement. For some this was a simple process of dividing a familiar even number by 2 – 22 feet became 11 inches – but others faced the challenge of finding half of 33 or 53. Again, decomposing these numbers into tens and ones proved an effective tool for supporting division.

Once the children have computed all of the scale measurements, they may begin mixing paint and natural materials to capture the color and texture of their soil layer. When each piece is dry, have the class work together to order their layers correctly and join them together, with the surface at the top. Our well log art was a 13' visual representation of a 300' cross-section of the earth under our school, which we proudly displayed in the classroom.

RESOURCES

Well logs are a matter of public record, and are available from the state Water Resources Department. If you do not have a well at school or at home, you should be able to request a well log from your community to analyze.


(12) WELL LOG:
Ground Elevation

Material	From	To	SWL
Soil Brown Sandy	0	3	
Clay Brown	3	20	
Sand Brown Fine Clayish	20	29	
Clay Brown	29	59	
Clay Gray Sandy Soft	59	78	
Clay Gray Sandy	78	87	
Clay Gray	87	135	
Clay Gray Sandy	135	168	
Clay Gray & Brown	168	221	
Clay Gray & Brown gritty	221		
w/Sand Course Multicolor		228	
Clay Gray	228	280	
Shale Brown Fractured	280		
w/Sand course-fine black		286	103
Basalt Gray & Brown	286	289	
Basalt Gray & Brown por	-289	305	103
Gravel Black Small w/	305		
Sand course-fine Black			
& wood		307	

Date started 10-12-94 Completed 10-14-94

FOREIGN OPERATIONS
INTERMEDIATES INVENT ALTERNATE BASE SYSTEMS

by Linus Rollman

The power, beauty, and mystery of mathematics are sometimes obscured for students and teachers alike, both by the frustrations that many people experience around math and by the sheer familiarity of daily experience. Our system for noting numbers and making basic calculations, which is the fruit of thousands of years of labor conducted around the globe, often seems in particular to lack romance. It can be hard to get really excited about the method that we use for comparing prices at the grocery store or balancing a checkbook. We take our tidy system of columns and decimals for granted. This lesson string requires students to move beyond the comfort of everyday counting and calculation by creating their own base system and learning to perform the four basic mathematical operations in it. In so doing, students not only build their problem-solving skills, but enrich their understanding of the base ten system. This project is a bracing antidote to rote memorization of procedures – in order to learn to "carry" or "borrow" in another base system, a student needs to think very carefully about what it means to "carry" or "borrow" in base ten.

These lessons have worked well with fifth-grade students, but are intriguing for older students as well. I would hesitate to use them with students much younger than fifth grade as they require real mastery of all four operations in base ten. The structure of the lessons is flexible. The most important thing is that the students have plenty of time to work and explore independently, as well as with your help and support. The lessons require no materials other than pencils and paper and perhaps calculators, but they do demand a good deal of time and patience and a willingness to try something new and difficult. The string is divided into five lessons; you might choose different break points depending on the abilities of your students. At Arbor, we teach this string with several days between the lessons during which the kids have time to work independently or in small groups on the project.

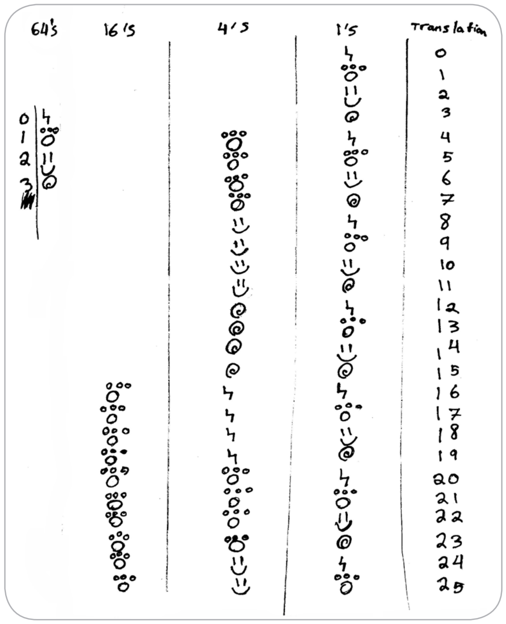
I strongly recommend that you invent and test your own base system before starting this project. Follow the steps that are outlined below for the students. There is no substitute for the depth of understanding you will achieve if you struggle with the same difficulties your students will undoubtedly encounter along the way.

Field Notes:
It is important to spend time examining base ten numbers, but it's also possible to lose kids' interest. Choose a moment fairly early in the lesson to announce that they will be creating their own base systems.

The Romans did not use place value to record numbers; their system was purely additive. In order to represent three, they wrote three ones in succession: III. They had separate symbols for five (V), ten (X), fifty (L), and so on. A smaller number written before a larger one indicated subtraction: IV was "five less one" or four. Seventy-nine was written LXXIX.

FORECAST
Lesson 1:
Introduce the students to the project. Begin by reviewing how the base ten system works. Ask volunteers to count up from one, each coming up to the board and writing the number that he or she has counted. When the students reach the number ten, stop and point out to them that something remarkable has happened. Ask them to explain how this number is different from the others they have written and how the number works: lead them to the idea that the 1 stands for one group of ten and the 0 for zero groups of one. Explore a few other numbers, into the hundreds and thousands, asking the kids to explain what the numerals signify.

Explain that this extremely useful way of writing numbers has not always existed. Give them one or two examples of Roman numerals. Show them that the Romans had a very easy way of writing 1000 (M), but that if you wanted to write 1984, you had to write MCMLXXXIV. Help them imagine that adding and subtracting, let alone multiplying and dividing, in Roman numerals was something of a nightmare. Explain that, even when place value is used, ten is not the only choice. The Babylonians used base sixty, the Mayans base twenty. Ask them why they think that ten is the base we use. (There is no definitive answer to this question. Some scholars believe that it is because we have ten fingers.) Emphasize the importance of the zero as a placeholder and explain that people have not always used it.



The students can now begin creating their own base system, using any base up to 15 (but not 10). Each student should be allowed to choose his or her base, but you might caution them that the higher base systems will be more complicated to record. Their first job is to invent symbols for the digits that they have chosen. Any symbols may be used, but point out that they will be writing these symbols often, so relative simplicity is a good idea. Have the kids arrange their symbols in a place value chart with base ten translations beside them. The first column is for the ones' place, the next for whatever value they have chosen for their base system (in base five, it would be the fives' column). Do not tell them what the next column should be. Allow them to discover this as they fill in their charts.

Historians of math disagree on where and when the zero was invented. It might be interesting to point out to the kids that historians have a tendency to attribute such important inventions to groups with which they feel cultural alignment.

Most kids should reach at least the third column of their chart, but this goal may not be realistic for those who have chosen high bases – the third column in a base fifteen system is the two hundred twenty-fives' column. This will require plenty of independent work time.

Lesson Two:

Begin with a discussion of the work from last lesson. Ask them to share their base systems with one another. What do the different systems have in common? In particular, how does one determine what the third (and fourth, and fifth, and so on) column should be? Give them some base ten numbers to translate into their own notation systems. Ask them to discuss their translation strategies with a partner.

Now it is time to learn to add in their base systems. Have them explain the process of addition in base ten using an example problem in which "carrying" will be necessary – say, 17 + 16. In particular, ask them to explain the process of carrying. Why is it necessary to carry? How does it work? What, exactly, is being carried? If tens are carried in base ten, what do they think will be carried in their own systems?

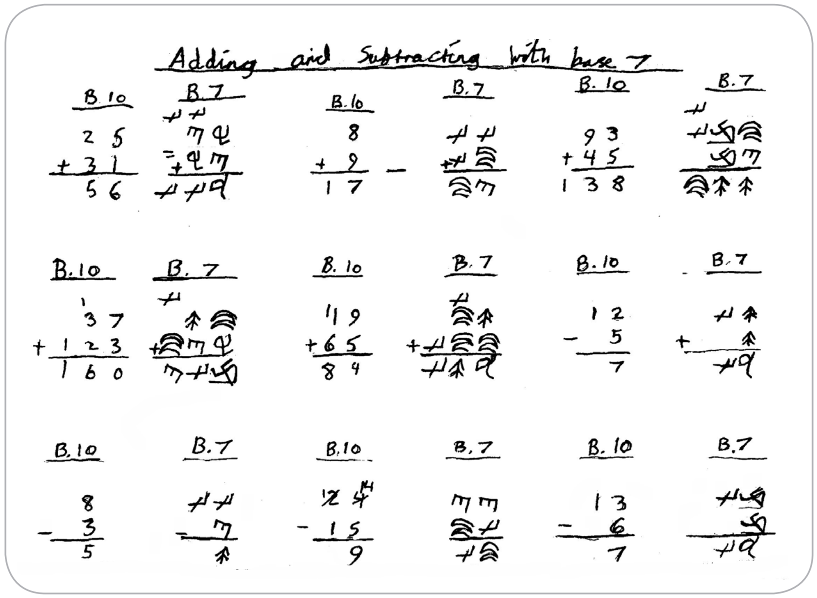
Give the kids time to explore the process of addition in their own base systems. They should create their own problems, writing them in base ten, translating the numbers into their own systems, trying the problems and checking their work by translating their answers back into base ten and doing the original problem in base ten. Encourage them to work with a partner or small group during the process, especially if they find themselves getting stuck.

As you work with individual students, make sure that they are giving themselves problems to try that involve carrying. Because of the novel symbols they are using, it can be difficult for both students and you to track their work. They should provide base ten translations next to each problem that they try.

At the end of their work time, group the students in pairs or small groups and have them teach one another how to add in their respective base systems. Close with a group discussion in which you ask students to show what they've learned by doing a few problems on the board.

Lesson Three:

This lesson, in which the students learn to subtract in their own base systems, can be structured more or less identically to the addition lesson. Begin with an example in base ten for them to analyze. In this case, the aspect of subtraction that needs to be emphasized and explored is "borrowing." When is it necessary to borrow? What is borrowed? How will borrowing look in their systems? Give them plenty of time for exploration and chances to help one another. To close the lesson, ask them to teach a partner how to subtract and, again, give them a chance to share their new skills with the whole group.



Lesson Four:

The next step is to learn to multiply. We are able to multiply in base ten largely because we have a certain number of basic multiplication facts memorized. It is not realistic for the students to memorize multiplication facts for their own systems. Instead, have them construct multiplication tables to refer to as they try problems. These tables should be at least five by five for a base five system, eight by eight for a base eight system, and so on.

Once the students have filled in their tables, they can try larger multiplication problems. This is harder than adding or subtracting and will require plenty of independent and scaffolded work. Structurally, this lesson can be handled as the last two were. I recommend carefully illustrating how multiplication in base ten works before the students begin working in earnest on their own systems. Write out the results of each individual multiplication and add those, rather than "carrying" from one column to the next:

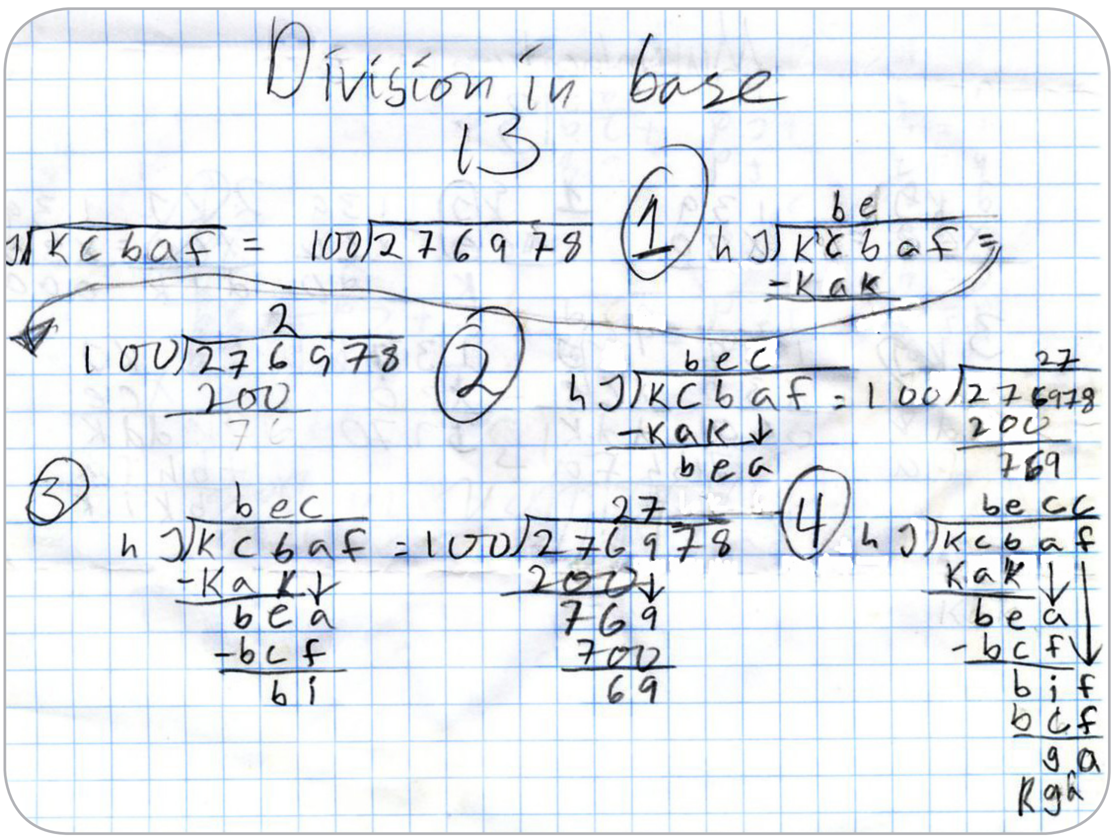
34		^{2}34
x 25		x 25
20	rather than	170
150		680
80		850
600		
850		

If your students are not used to the first recording method, you should take the time to make them comfortable with it in base ten, since it will help clarify their work in their own systems. As with addition and subtraction, close by asking the kids to teach one another to multiply in their base systems.

Some students may be tempted to simply do each calculation in base ten and translate it, digit by digit, into their own system. Make sure that calculations are actually done in their own base systems, and that the corresponding base ten calculations are done only to check their work.

Lesson Five:

Division is the hardest operation of all. I recommend attempting it only if your students are enthusiastic about the project. You might make this lesson an optional extension for those who have worked most quickly through the first three operations. Again, it is best to illustrate a division problem in base ten, step by step. Division involves making estimates and multiplying, so the students' multiplication charts from the last lesson will be necessary tools.



Culminating work:

Have the kids create posters explaining their base systems. Your students might present and teach their systems to their parents at an event or celebration.

Alternate base calculations by Ursula Clausing-Hufford, Will Glisson, Mira Reichman, and Alex Lam.

RAISING TO A POWER
SNEAK PEEK AT *AN INTRODUCTION TO ALGEBRA*

by Linus Rollman

At Arbor, we rarely teach by the book. Just as we train our students to do in their research work, we compile large numbers of books, use the most relevant parts, and throw in material of our own to make it all as engaging as possible. In Senior math, we found ourselves deconstructing three different textbooks and reassembling an amalgamation, supplementing with logic puzzles, handwritten work packets and tests, and math history from a variety of civilizations. The goal is always for our middle schoolers to become literate and comfortable in the language of algebra before they leave us to enter high school.

Our Senior math teachers have encouraged natural and fluid patterns of work in the classroom. Students move through the curriculum at a largely self-determined pace; some choose to work individually, but most settle into small groups with peers working at a similar rate. They do their problem-solving in composition notebooks that teachers can assess periodically, rather than turning in a night's work on loose leaf for a grade and then discarding it. As they encounter new concepts, they are required to add to a personal textbook of Notes to Self, describing the new material in language that makes sense to them and including examples that will help them solve similar problems in the future. They may use their Note to Self books on every test. Their teachers circulate to check in, pose questions, encourage deeper thought, and sometimes to interrupt the flow of independent work to bring an interesting discovery or an innovative solution to the whole group's attention. In observing students' self-paced work in beginning algebra over the years, we have revised and reorganized the curriculum to introduce concepts in the most logical progression, to keep the problems approachable and fun, to solidify students' understanding of difficult ideas by asking them to author their own problems. We have encouraged self-reliance and teamwork, experimentation and multiple approaches, confidence and playfulness. The result looks like no other algebra curriculum we've encountered.

Tests do not determine grades at Arbor, but serve as assessment tools. We believe our open-book model is better representative of the problem-solving challenges our students will meet beyond the world of school, and that teaching them to build and rely on authentic resources is more valuable than asking them to memorize and recall in high-pressure situations at this young age.

The obvious next phase? To publish our own Introduction to Algebra. *In the capable hands and writerly brain of Linus Rollman, Senior math and humanities teacher, a summer's work has brought the goal in sight. After rigorous student testing during the 2008-09 school year, this sixth-grade algebra course will be available for purchase in 2009. Here is an excerpt from Chapter Two: Writing Algebra, introducing an advanced topic in the study of place value: exponents.*

- Sarah Pope, Publications Editor

Lesson 5: Exponents

1. **Imagine that you are the owner of a tribble farm. Tribbles are small, furry, made-up creatures that reproduce like amoebas – that is, one tribble splits in half to create two tribbles. Every tribble splits in half every hour. Let's say that, in the very first hour that your farm is operational, you have two tribbles. In the second hour, you will have four. How many tribbles will you have after a full day has gone past?**

   You might want to start out with drawings for this problem, but I think you'll find that the drawings become difficult pretty quickly.

In that problem, you found out that after a single day, you would be the proud owner of an extremely large number of tribbles. It would be convenient, wouldn't it, to have a shorter way of writing such a long number? Well, there is such a way, and that's the subject of this lesson.

In the first hour, you had just two tribbles. In the second hour, you had four, or two times two, tribbles. An expression for that number could be any of the following:

$$4 \text{ or } 2 \cdot 2 \text{ or } 2^2$$
$$4 = 2 \cdot 2 = 2^2$$

In the third expression, the little two on the upper right of the larger two is called an *exponent*. It is really just a bit of shorthand – a simple way of saying, "multiply two by itself."

In the third hour, you had eight, or two times two times two, tribbles. This can be written as follows:

$$8 \text{ or } 2 \cdot 2 \cdot 2 \text{ or } 2^3$$
$$8 = 2 \cdot 2 \cdot 2 = 2^3$$

Again, in the third expression, the little three is called the *exponent*. The two beside the little three is called the *base*. And what it means is, "multiply the base by itself exponent number of times," or, in this case, "multiply two by itself three times."

Now, it's probably not obvious yet why exponents are a useful idea. But let's see what happens when we get to hour 18. Now our three choices for how to express the number of tribbles look like this:

$$262{,}144 \text{ or } 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \text{ or } 2^{18}$$
$$262{,}144 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{18}$$

You can begin to see that the third option is more convenient than the second. As for comparing the third option with the first, that's really a matter of what particular situation you're in. Sometimes it might be more useful to write 262,144, and sometimes you might have reason to prefer 2^{18}. As I mentioned in the last chapter, part of getting skilled at algebra is getting used to expressing things in different forms rather than thinking of one side of an equal sign as "the answer."

Of course, the base does not always have to be two. You could write:

$$5^3 = 125$$
$$3375 = 15^3$$
$$49 = 7^2$$
$$18^7 = 612{,}220{,}032$$

One more piece of vocabulary and then I want you to work with some exponents. Using an exponent is also called *raising to a power*. The expression 6^7 can be read, "six raised to the power of seven" or "six to the seventh power."

Translate the following into mathematical symbols:

2. **five raised to the fourth power**
3. **ten to the power of five**
4. **fourteen raised to the third power**

5. **Which of the three numbers you just wrote do you think is the largest? Use your calculator to check.**

Express each of the following numbers as a power of the other number given (check with your teacher about whether you can use a calculator for these problems):

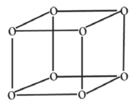
6. **216 as a power of 6**
7. **3125 as a power of 5**
8. **10,000 as a power of 10**
9. **10,000,000 as a power of 10**
10. **6,561 as a power of 81**
11. **6,561 as a power of 9**
12. **6,561 as a power of 3**
13. **If 3^6 is 729, what's 3^7?**

There are a few other things to know about exponents. When you raise a number to the second power, it's also called *squaring* that number. So, 3^2 is read "three squared." There's a good reason for this. The multiplication problem it stands for is three times three. If you drew a picture to illustrate this multiplication problem, it might look like this:

o o o
o o o
o o o

14. **Make drawings to illustrate the multiplication problems that go with "four squared" and "five squared."**

When a number is raised to the third power, it's also called *cubing* the number. For example, 2^3 is read "two cubed." Again, there's a very good reason for this. An illustration that might go with "two cubed" – which, of course, means "two times two times two" – could be:



As you may have noticed from some of the problems that you just did, there is one number that, when used as the base of an exponent, has a special property. That's the number ten. We happen to count using a base ten system. What this means is that when we write numbers, we record them in groups of ten and multiples of ten. For instance, when we write the number "453," it means "4 groups of one hundred, 5 groups of ten, and 3 groups of one." Because of this fact, ten behaves in a particular way when you raise it to a power.

15. **What is 10^2?**
16. **What is 10^3?**
17. **What is 10^4?**
18. **Based on those examples, what do you think 10^{15} is?**
19. **How many zeros would you expect to follow the initial 1 in the number 10^{100}?**
20. **What is the advantage of writing 10^{100} instead of writing the expanded version with all of those zeros?**

The answer to that last question indicates something important about this quality of the number ten. I won't go into it right now, but this quality will be very useful when you learn how to use scientific notation, which is a short, convenient way for writing very large (or very small) numbers.

21. **Make a drawing to illustrate "three cubed."**

None of the powers after three have special names like squaring or cubing. Once again, there is a good reason for this.

22. **Why is it that there is no special word to go with raising something to the fourth power? You might try making a drawing of "two to the fourth power" to help yourself figure this out.**

And, of course, since we are working in algebra, there is always the possibility of using variables in expressions that have exponents. So, for example, these are all fine:

$$x^4 \qquad 7^y \qquad a^b$$

The first one is read "*x* to the fourth power," and it means, as you can probably easily figure out, "*x* times *x* times *x* times *x*" or "$x \bullet x \bullet x \bullet x$." The second one is read "seven to the power of *y*," and it means "seven times itself *y* times." The third one is "*a* to the power of *b*," and it means "*a* times itself *b* times."

Write expressions using exponents for each of the following:

23. **the number *x* squared**
24. ***m* raised to the fifth power**
25. ***m* raised to the *n*th power**
26. $y \cdot y \cdot y \cdot y \cdot y$
27. $\underbrace{4 \cdot 4 \cdot 4 \ldots \cdot 4}_{\text{m of them}}$

There is also no reason that exponents can't be combined with the other operations. You could have any of these:

$$x^3y^4 \qquad a^3 - b^6 \qquad 4x^3$$

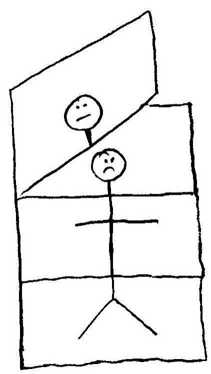
The first one is "x to the third power times y to the fourth power," and it means "x times x times x times y times y times y times y." The second is "a to the third power minus b to the sixth power." The third is "four times x to the third power," or "four x to the third," or "four x cubed," and it simply means "four times x times x times x," or "$x^3 + x^3 + x^3 + x^3$."

Write expressions using exponents for the following:

28. ***m*** **to the fifth minus** ***n*** **to the fourth**
29. **3 raised to the power of** ***a*** **plus 5 raised to the power of** ***b***
30. ***y*** **cubed times** ***z*** **squared**

31. **This is a pretty challenging problem – one that you will probably want to work on with other people – first presented by the mathematician Harold Jacobs in his book *Mathematics: A Human Endeavor*. It involves exponents and it is one of my favorite problems in the world because the idea behind the book that's in the problem is so amazingly cool.**

    A French gentleman named Raymond Queneau wrote a book called *Cent mille milliards de poèmes*. The title means "one hundred thousand billion poems," and he called it that because *that's how many poems it actually contains!* Here's how it works: Every poem is fourteen lines long and each page is cut into fourteen strips with one line on each. So you can turn the individual lines of the poems as separate pages, mixing and matching them. It's really just like those books you probably saw as a kid where the pages were divided into three: the tops of the pages had the heads of creatures on them, the middles had torsos, and the bottoms had legs, and you could flip through the segments to create different, weird creatures.



    (By the way, the pages only have pictures -- or lines of poetry -- on one side.) The question for you to answer is, "How many pages did Queneau's book have to have in order to make a hundred thousand billion different fourteen-line poems?"

Here are some bits of advice:

Work with some simpler cases first. Say, one of those creature books where the pages are cut into three that has maybe five pages. How many creatures could you make? What if it had four pages? Or six? What if it were cut into two strips or four strips instead of three? How are exponents involved in these questions?

Drawings or charts or tables will probably help you figure out the creature books I just mentioned.

Now try working with the fourteen-strip book that produces one hundred thousand billion poems. Write "one hundred thousand billion" in numbers instead of words. Then try writing it as a number with an exponent: specifically, how could you express it as a power of ten?

32. **Last thing for this lesson: a Note to Self about exponents. As always, the goal is to record what exponents mean simply and efficiently and to give several examples of how they are used.**

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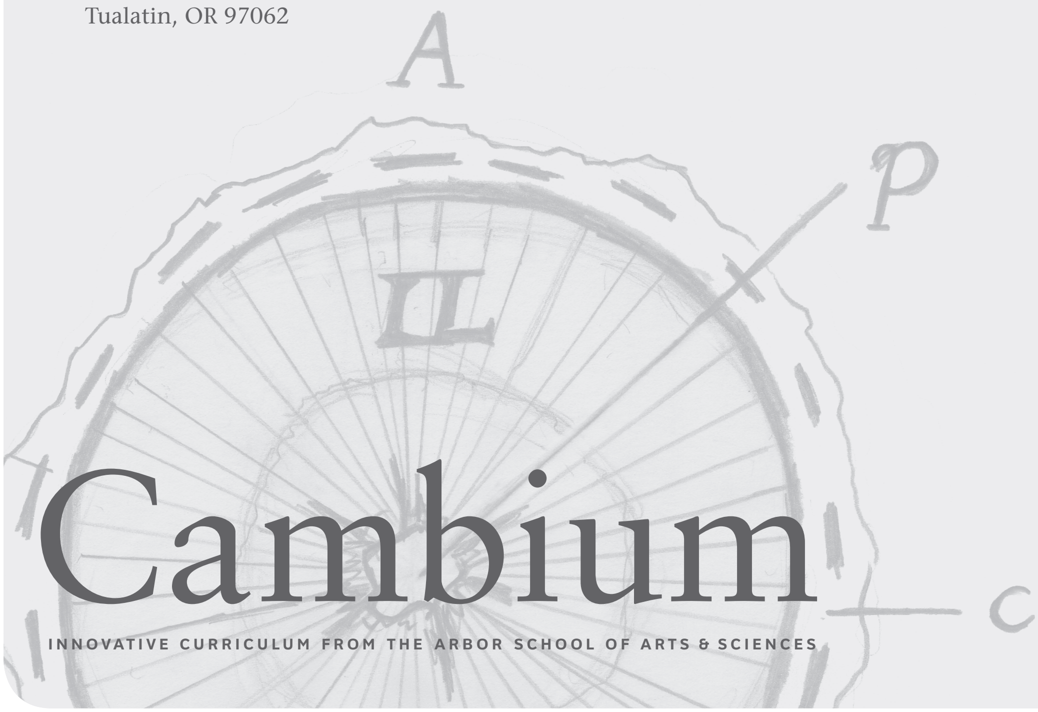

Cambium

INNOVATIVE CURRICULUM FROM THE ARBOR SCHOOL OF ARTS & SCIENCES

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Cambium: (n) the cellular growth tissue of trees and other woody plants, from medieval Latin "change; exchange."

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Masthead by Jake Grant, after an 1890 botanical illustration. Plant block print by Annika Lovestrand.

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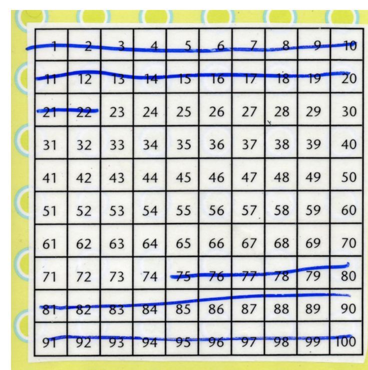

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