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A Sensible Practice for Sense-Making

The simplicity of introducing a Number Talks routine for deepening quantitative reasoning

The depressing thing about arithmetic badly taught is that it destroys a child's intellect, and, to some extent, his integrity. Before they are taught arithmetic, children will not give their assent to utter nonsense; afterwards, they will.

W. W. Sawyer, 1961 – as cited in *Making Number Talks Matter*

One of the most exciting things about mathematics is that it works. Whether it is seen as an innate system laced throughout the natural world and discovered slowly by mankind, or as humanity's own invention for approximating descriptions of relationships in nature, all mathematical principles share extensive, interconnected roots that hold true. As students study the foundational properties of arithmetic and conventions of mathematical notation, they are working within a thorough system of logic. Operations can be reversed and figures can be inverted to describe the relationships from alternative perspectives – processes that are often necessary when working to identify unknown quantities, as we do in algebra.

New to teaching last year, I was introduced to the Common Core's eight Standards for Mathematical Practice. The very first standard requires that students "make sense of problems and persevere in solving them." Though I'd long known that even very intelligent people can be confused by math, that anyone would not try to make sense of a problem was wholly new to me. For any trouble that I might have had keeping straight the details of calculus, I always expected it to make sense.

As it turns out, faith in the integrity of math is not universal. Some students arrive expecting math to make sense, and will put great thought into a confusing concept until it does. Others may concentrate their efforts in memorization, holding on with a sort of blind faith. There are also non-believers, disinterested in the subject and frustrated by the long list of rules handed from on high. As students develop understanding of the reliable ways quantities relate, we say they are developing number sense. A solidly developed number sense is like a moral compass in the faith, a literal compass upon the terrain, and is essential for arriving at true mastery of the concepts we aim to teach.

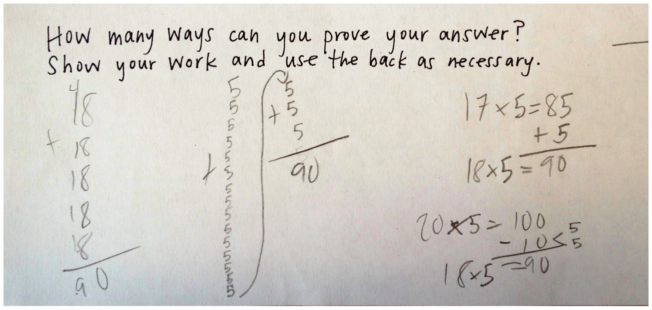
The Number Talks method was developed by a few teachers in the 1990s, with the goal to help students "take back the authority of their own reasoning." (Humphreys and Parker, *Making Number Talks Matter*) Many documents and several publications on Number Talks have resulted from practice at multiple grade levels and incubation amongst graduate students and other educators. On the website of Math Perspectives (a teacher development center headed by Kathy Richardson, one of teachers credited for creating the Number Talk), it is described as "a short, ongoing daily routine that provides students with meaningful ongoing practice with computation." The exercises may range from single operations with familiar numbers to complex problem scenarios that lead to estimation practice and experimentation.

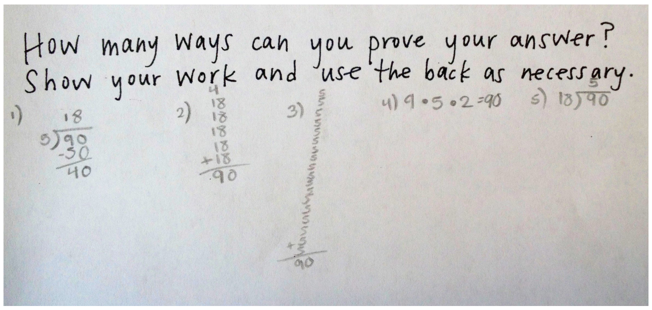
Though there are a few variations on the Number Talks recipe, most recommend spending only about 10 minutes and keeping the computations mental. A problem is presented on the board, and students are expected to think it through strategically, rather than to lean on the familiar algorithms that are less efficient without the aid of pencil and paper. Students give a discreet thumbs-up signal to the teacher when they've arrived at a solution, so as not to distract others who have yet to do so. After finding a solution, students are implored to consider other possible approaches, further testing the flexibility and strength of their number sense. Additional solutions are indicated with additional fingers. When most students indicate having found a solution (or a few minutes have passed), volunteers are requested to share possible answers, and then to share strategies with the class. In turn, students relate their processes step by step, and the teacher interprets this process in writing on the board. Students are encouraged to ask questions and discuss one another's strategies, particularly if there remains disagreement about the final answer. Some practitioners recommend naming and collecting all strategies on a permanent public record, or giving students another problem on which to practice the new strategies immediately after the talk.

Number Talk Protocol

- **students** display readiness with a closed fist at the chest; **teacher** writes a problem on the board
- **students** solve the problem mentally, indicating readiness with a thumbs-up; **teacher** allows wait time
- **students** share possible answers; **teacher** creates list on board
- **students** share strategies step by step; **teacher** charts thinking on board

Some also recommend allowing students to pair-and-share before strategies are collected, giving the teacher time to circulate and select appropriate ideas to share with the group, or the license to cold-call after students have each had a chance for private discussion. I personally have struggled to collect information when many different conversations are happening, but allowing students to pair-and-share can help an uncertain student to articulate her idea and thereby develop confidence to share with the whole group. On the other hand, even a two-person discussion may deprive one of the opportunity to think things out for oneself. Alternating teaching methods—between those that invite collaboration and those that encourage personal contemplation—is perhaps the true ideal. The basic format of Number Talks is so simple that there is room for any teacher to personalize according to daily objective and student needs.





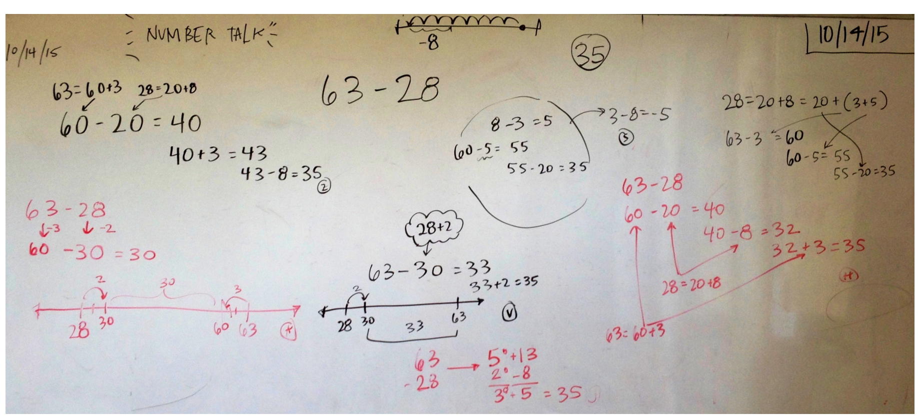
Repeated addition, though an important concept to understand in the mathematical landscape, is hardly an efficient strategy when not combined with other mathematical principles and properties. On a written pre-assessment, this was a popular alternative to the algorithm, which most students used first.

I have been practicing Number Talks regularly

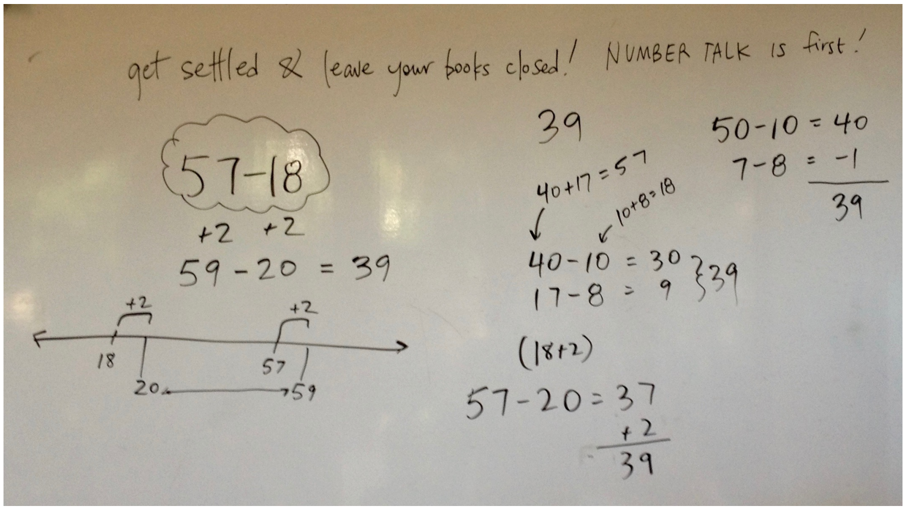
I have been practicing Number Talks regularly with a class of seventh graders at Arbor, the small independent school hosting my teaching apprenticeship, and bringing them into sixth and eighth grade classrooms occasionally as well. The first day of Number Talks for each group took some by surprise. “What do you mean ‘different ways?’” they’d inquire. Many were adept enough to offer some alternative problem-solving approaches, but few of these were models of efficiency. It’s important to know that multiplication can be thought of as repeated addition, but is adding up 18 individual 5s going to be as useful or efficient as multiplying ten 5s and then adding that to another eight?

I started the term most interested in encouraging students to think outside of the algorithms with which they’ve become familiar, and which most depend on for comfort and accuracy. When faced with a double-digit multiplication problem, myself, the old reliable algorithm had been my go-to, even mentally. Though I was always a believer in the truth of mathematics and the availability of alternate routes through the landscape, I still kept to the well-trod path, leaving roads less traveled to become overgrown. Even a shortcut can seem laborious when untended! While we do want them using these in their written computations, as they are effective and efficient in most situations, over-reliance without deep understanding can be problematic. Too often a mechanical error within the tightly-packed machine of the algorithm will lead to a very wrong answer with reasonability unchecked. Students should possess understanding of their tools; the deeper understanding, however, is more difficult to assess than is execution of the tool itself. In the service of stronger number sense, I have tried to push the students to be more flexible in their thinking, and to practice articulating this thinking clearly to others.

Here, the sixth graders talked through different ways to subtract 28 from 63. Several versions of what the Number Talks handbook calls “round and adjust” can be seen, as well as breaking the numbers apart by place value.



Our first Number Talks involved straightforward multiplication and subtraction work that could invite multiple strategies. Even in a simple subtraction problem, some students found elegant strategies not considered by the majority of their classmates. In the seventh grade class's first subtraction Number Talk, the last strategy shared was one that the Number Talks literature refers to as "same difference." If the minuend (the number being subtracted) can be rounded to a more "friendly" number - like 18 up to 20 - the other number (the subtrahend) can be adjusted by the same amount, and the resulting subtraction problem will yield the same difference as the original. Several other students in the class reacted audibly to this demonstration, to the effect of "Wow, that really was easy!"



In my first attempt to encourage the "same difference" strategy brought forward in a previous Number Talk, another student took the bait! But when I later offered a subtraction problem with fractions, the type students tend to have more trouble with, no one tried it.

In several subsequent Number Talks I tried to invite further exploration of this strategy by using numbers that could be easily rounded. As decimal and fraction operations tend to give students more trouble than do whole numbers, I hoped that students would extend this reasoning to those arenas. Some students applied it to decimal subtraction, but no one used it for fractions. After several days of untaken baits, despite the lingering potential, I realized it was time to move on. I still believe the practice was meaningful, but another tenet of Number Talks is that, though the routine itself is to be reliable, the talks "should never be predictable for the children." I was unable to predict the strategies they would pick, but that I would choose a subtraction problem for Number Talk was starting to feel like a given.

Moving on, we spent several Number Talks comparing fractions. Merely conceptualizing these figures, it turns out, involved multiple calculations – the better to push students toward efficiency – and often made room for spatial reasoning, too. I found out by listening to my students that what I see as simple prompts can still provide interesting territory for exploration. On the other hand, the subtle differences in methods that I find so interesting will not necessarily be what stands out to the students. With no set curriculum to follow, I've sought Number Talk material that will require thinking that students aren't used to doing, while still allowing multiple routes in, as mathematics always does. Over the course of our fall term Number Talks, there were several opportunities for elegant strategies to arise – new tools that might have lain dormant and unexamined if not given the platform of open Number Talk.

A few months into the year I took an informal survey of some students both at Arbor and at a local public school where I observe: “How many of you expect math to make sense?” Sure enough, only about half of the students in each group said yes. Further questions on my survey methods aside, this awoke me to the reality that I myself had persisted in doubting.

At the Arbor School, middle school students begin the algebra sequence with a unit on reasoning, both inductive and deductive. We aim to ground them in a sense of ownership of knowledge, with a metacognitive skill set to consider how they know what they know. Still, given the above question, not all were ready to admit confidence in this logic. We were already well underway with Number Talks, too, but rather than feel disappointment, I gained renewed interest in finding problems to exercise their sense-making muscles. Uncertainty may not be a bad starting point if students can come to depend on math through their own explorations. Experiential learning in math leads to further faith in logical systems, logically. Unfamiliar problem contexts—ones for which there are no handy algorithms—best lend themselves to this work. As Humphreys and Parker write in *Making Number Talks Matter*, “cognitive dissonance is a valuable and even necessary part of the process of learning.”

I myself had delighted in the many different routes through the littlest problems, and was initially worried about overwhelming my students with complicated tasks. But at the prompting of my mentor Annmarie, I began pulling more complex problems from the website of another Number Talk practitioner (Fawn Nguyen, mathtalks.net); these, contrary to my fears, have provided some of our most interesting discussions. And when it has seemed that very few students were making headway on their own, these have presented opportunities for pair-and-share. Ideally, through this sharing, students can give each other hints—clear the way a bit for further progress.

One of the most successful Number Talks, from my perspective, came in late October, with an invitation for reasoning that needn’t necessarily involve calculations. I asked the seventh graders, “Which is greater, 79 x 25 or 29 x 75?” The freedom to estimate brought out a new kind of Number Talk. Some less-frequent contributors articulated their rounding strategies, each of which produced a slightly different estimate but pointed to the same pair of factors having the greater product. The last two students I called upon were ones I had expected to keep their hands up until the end. Neal spoke first.

“I’m not really sure how to describe it,” he began, which statement by itself was exciting. Though quiet and not often heard in Number Talks, Neal is known by his peers to have a strong understanding of math, and so his admission of uncertainty was doubly significant. He went on to

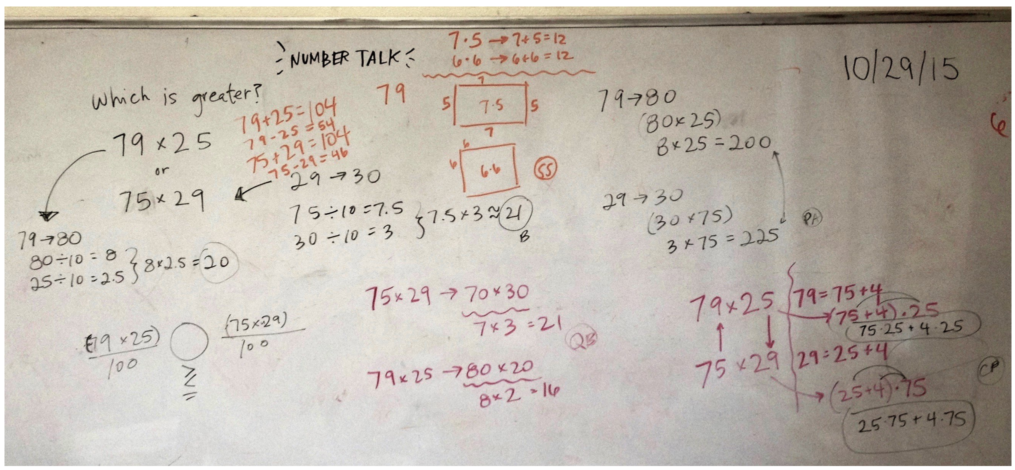
Common Core Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

read more about the practice standards at www.corestandards.org/Math/Practice/

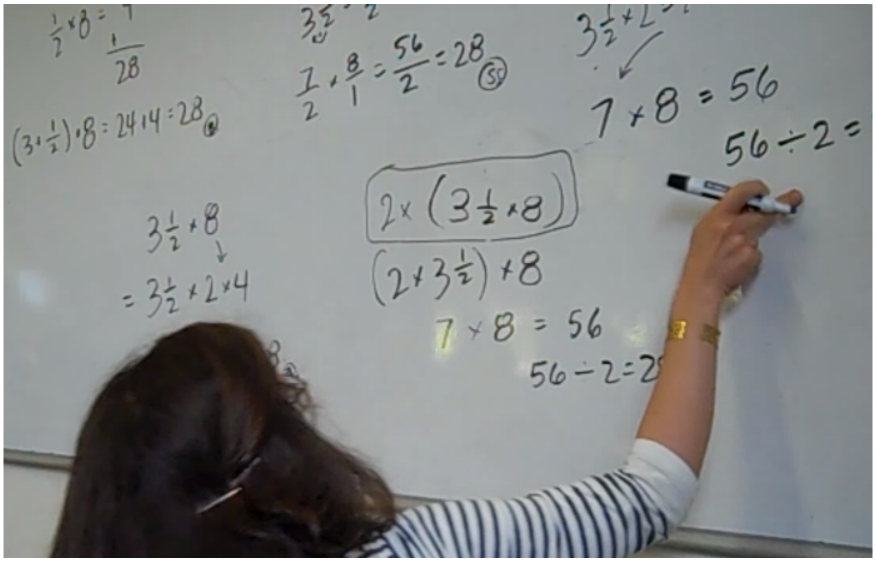
say that he saw the difference between the two expressions, 79 x 25 and 29 x 75, as nearly the same but “with an extra 4” – one “in the 79” and one “in the 29”. What he admitted trouble articulating was what made the 29 “worth more,” so I took the extra step of breaking those two factors into addends – (75 + 4) x 25 and 75 x (25 + 4). From here, it was a little easier to prove that distributing the 4 produced extra 25s in the first case and extra 75s in the second.

When Jeff’s turn came, he too admitted an inability to explain the faith he had in his own reasoning. “I just know that when you multiply two pairs of numbers that add up to the same thing, the pair that are closer together will have a larger product.” It took some time for me—and probably for his peers—just to comprehend this initial claim. I pointed at the two pairs of numbers, agreeing that the sum of 25 and 79 was equal to the sum of 75 and 29, as suggested by Neal’s separation of the “extra 4.” I asked if anyone else had noticed this pattern before, but the room was silent. I suggested that we could all—outside of our limited Number Talk time—explore its plausibility through tests to arrive at the same level of certainty that Jeff had, but then it occurred to me to draw arrays. Rectangles of the dimensions 25 x 79 and 75 x 29 will have the same perimeter, corresponding to Jeff’s claim about sums. As we could more quickly test with smaller numbers, the area of a rectangle with a given perimeter is optimized when it is a square: 8 x 4 and 7 x 5 yield lesser areas than does 6 x 6. What each of these students believed to be true through faith in their own strong number sense was ultimately made provable with careful communication.



At lower right, I attempted to write out what Neal described about the “extra 4,” and arrived at a familiar form of expression for use with the Distributive Property. At top center, I wrote a few equations to illustrate Jeff's points, before turning to the visual models - two rectangles with equal perimeter but unequal areas.
In hindsight, I might have written out more intermediary steps for both strategies — but as with any live translation work, precisely what needs to be communicated can be difficult to determine!

Communication is a key focus in all realms at our school, and the authors of the Common Core have likewise indicated the importance of communication in math. The third Common Core Practice Standard calls upon students to construct and critique mathematical arguments, leaning on foundational logic to communicate ideas effectively. In our flipped classroom, most students work in step with a partner or group, and class time affords opportunities to talk about the material with both teachers and peers. But as we invite students to work at a self-directed pace,



Here, I unpack Mike's approach to 3 1/2 x 8: multiplying everything by 2 at the start to eliminate the fraction, and then dividing by 2 for his final answer. Another teacher spoke up, asking whether he meant that he'd taken that 2 from the 8, using factoring and the Associative Property of Multiplication. After I wrote that strategy out, too (left), he responded hesitatingly, "No, that's not what I did." From there, I returned to my original, direct interpretation of Mike's words, with the determination to show his actual steps more clearly.

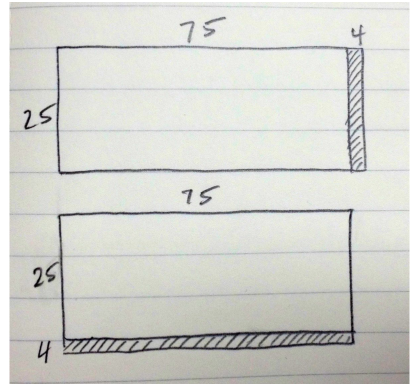
some students wind up without partnerships. Even when those students are successful at independently digesting concepts, they miss out on the regular peer communication that we value in the math classroom. For all students but especially for these, the Number Talks routine has helped carve out daily time for this practice as well.

As teacher, I also get communication practice from a Number Talk. The exercise is in listening, with a focus on students' developing ideas. With cautious, literal interpretation and transcription of student arguments, I hone my listening skills while modeling mathematical records on the board, encouraging students to be explicit and precise in their language. I suggest clarifications—"Is this what you mean?"—when the student cannot, but for the most part keep myself bound to their words, both empowering them as sharers and teachers and holding them accountable for precise language. Attending to precision is yet another Common Core Standard for Mathematical Practice (6), and is as applicable to communication as it is to computation.

Nearly every day in Number Talk, a few familiar hands are the first up and, if I stay true to my goal to wait for and prioritize others, those hands tend to stay up until the end. These aren't necessarily the most "advanced" as measured by the curriculum: two of my most reliable and creative Number Talkers are behind most of the class in our algebra textbooks. But while not always focused and self-driven in our flipped classroom, these students have shown themselves to have incredibly flexible thinking in the math landscape—solid number sense. For these students, Number Talks have been both an outlet for their exercise and an alternative assessment for us teachers.

The Number Talks pedagogy recommends frequently integrating visual aids into students' offered strategies. I have at times made good use of this, but other times draw blanks, or run out of room on the board. Later, it will strike me: "oh, I could have shown it like that!" Neal's product comparison strategy, for example, could have just as easily been drawn in arrays, and may have been better understood by his classmates this way. While inefficient for regular computational reliance, visual models in an argument combine several more of the Common Core Practice Standards – noticing and making use of structure (7), and strategically using tools (5) for the contextualization of reasoning (2). I believe these to be worthy standards, and so must keep working to exercise and model them myself.

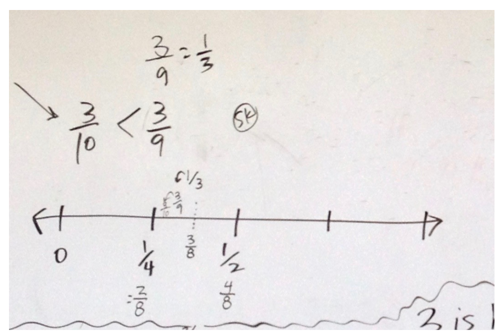
While simple memorization of rules and procedures and even practice standards can leave students without bearing in the landscape, highlighting student strategies in front of the group is a way to "teach children to be learners," a tenet of my school's founding philosophies. (Hawkins, *The Idea of Arbor School*). Such sharing not only demonstrates the specific strategies, but ideally sheds light on the process of constructing them.



Even without being perfectly to scale, a drawing can be helpful for comparing quantities. Here, the "extra 4" is revealed to be "worth more" when added to the 25, and therefore multiplied by the 75, a larger number.

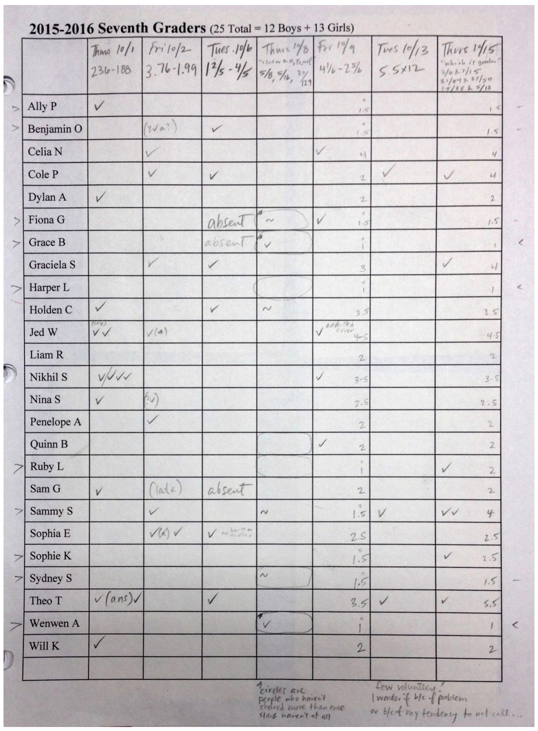
Occasionally, a student's sharing includes a mistake. A shyer student spoke up one day, and in her calculation was an assertion of something untrue, a mis-remembered math fact, something like 15 – 7 = 11. I racked my brain for the slip – could she have read one number as another? used the wrong operation? inverted the problem? There was no clear view into her thought process, so I was halted in my tracks. I said something like "huh, well, is that right?" and though she seemed to almost take the point, she was sufficiently confused as to then insist that her original claim was fine. I was moved to correct it in the moment—"No, I think, I mean, 15 minus 7 is 8," I managed to say, but not even sure of the proper way to say it publicly, politely. I felt that the student might benefit from remediation on a number line, but that that might be counterproductive in front of the entire seventh grade class. I certainly did not want to discourage her from sharing again.

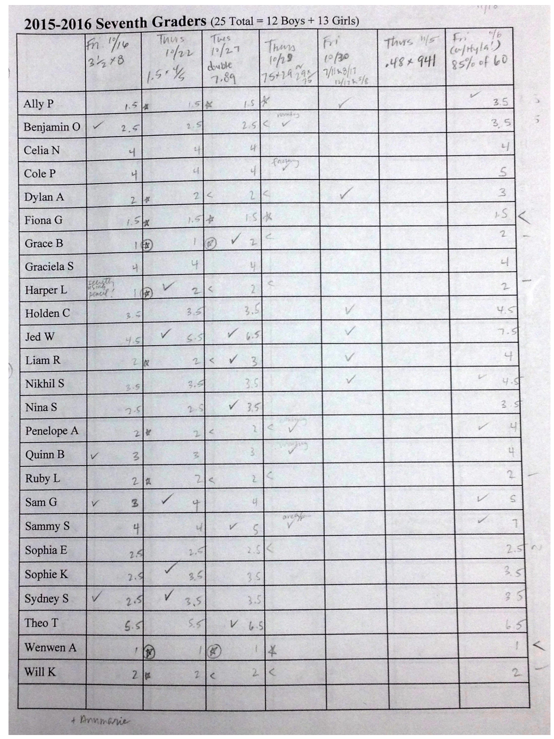
At other times, visualizations have been helpful to think through misconceptions: Derek's faulty assertion that one fraction would be larger than another because, though they were smaller pieces, there were more of them; or Anna's imprecise claim that 1/3 is half-way between 1/4 and 1/2. Our group is lucky to have some individuals like Wes, confident enough to talk through incorrect solutions, explaining his procedure until the mistake or misconception is revealed. Sometimes simply finishing the job of listening and recording without judgment can give the student an opportunity to say, "Wait, that's not right. I guess that changes my answer!"



Though it's true that 3/10 is less than 1/3, Anna's assertion that 1/3 was "halfway" between 1/4 and 1/2 was not accurate. A number line can help make that clear by showing what *is* halfway between the two benchmarks.

The primary problem I've encountered in our Number Talks is the imbalance of participation. Though participants are fairly representative of the spread of the class, I do have more male regulars than female. I have been keeping tally of student participation, and from our first weeks on I began bringing a list of the least-vocal students to the board with me, in order to prioritize my selection of sharers. This has helped to ensure that I call on less-frequent contributors when they do volunteer, though they may stand out to me less than the three quiet girls I'll never forget to look for. In this way, I have been successful at turning some quieter but more confident students into frequent volunteers.



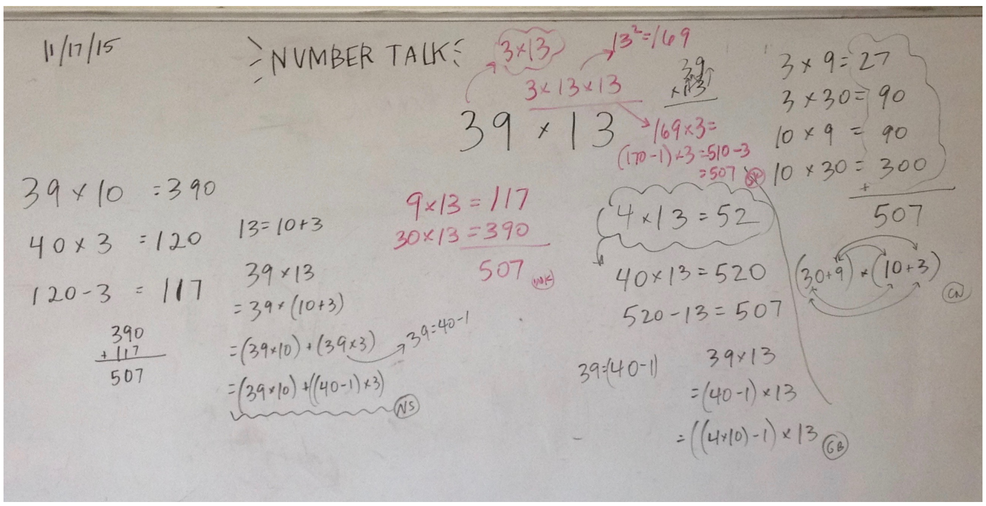


On a class list of students, I check off those who share each day. I began with separate marks for contributing to discussion or volunteering an answer at the start; but as the term progressed, I focused on tracking which students were actually sharing strategies. At least once a week, I total participation to prioritize who I will call on. While some persist in keeping quiet, I want to be sure to catch those in the second-least-vocal group when their hands do go up!

A few students still hardly ever volunteer, though I week after week tried to find problems that would invite them in. While some repetition of problem-type (including subtraction problems that invite the "same difference" strategy, discussed above) is generally encouraged in a Number Talks practice to allow use of the new strategies that arose the first time, I admit that it was in such revisiting that I expected to see my shyer students finally speak up. In prolonging my hopes for this, however, I not only failed to incur their regular participation but feel that I was not as adventurous as needed to make the most of our Number Talks.

As mentioned above, pair-and-share is a popular teaching strategy for exercising the voices and bolstering the confidence of those less inclined to share with the group. But I have reservations, fearing that it can be as discouraging as encouraging. Once, when eight slightly different estimates were given for a multi-step multiplication problem, I asked students to pair-and-share to discuss what were clearly many different approaches to the estimation. When I called the whole group back together to share, my only volunteers were those I was used to seeing every day, and each that I called on spoke in defense of more precise answers. Though I had intended to encourage thinking about the different ways to estimate efficiently, I felt that instead students with less precise strategies had been filtered out as “wrong.”

In another attempt to give less vocal students an opportunity to prepare, I asked the class to write down their strategies - but only after the usual wait time for mental calculation had passed. I selected the sort of double-digit multiplication that we had already addressed in Number Talks, thinking that these notes would offer a comparative point of assessment for me. Though not a new challenge, the writing exercise was fruitful, as it brought forward a multi-step abstraction from Lena (at that point tied for least-frequent contributor!) as well as a thorough breakdown of the traditional algorithm from Jess (not the very quietest, but one seemingly reluctant to try new approaches).



At top right, Jess’s approach makes use of the Distributive Property, carefully breaking apart the factors to get four partial products, while maintaining the place values obscured in the popular algorithm.
At lower right, Lena rounds the 39 to 40, and further simplifies it to 4 before calculating, then making adjustments for her final answer.

Several Number Talks have offered opportunities to unpack algorithms as a class, while demonstrating other ways to arrive at the same answers. But some students seem to remain stuck on the algorithm as their preferred solution path. To develop strong number sense, students need more than a directive to ignore the algorithm that could do the trick. They also need opportunities to think about problems they don’t already have methods for, to blaze trails where there are none. “I’m not sure how to explain it, but…” is a great sign of growth! Once again, this calls for Number Talks that provide novel challenges.

As a teacher in training, I had many motivations for exploring Number Talks. I certainly wanted to foster that number sense that we find so essential for mathematical fluency, and to give students the opportunity to practice making and hearing mathematical arguments. I also wanted to experiment with a routine that I could adapt to other future classes, something applicable at all levels, since I don’t know where I’ll be teaching when my program ends. The explicit role I have in Number Talks has been helpful to me in my present position, as well. I had had trouble finding confidence as a new teacher, particularly as we work with a flipped classroom model, students leading their own learning through the books. Even in a student-centric learning environment, the teacher must be respected and trusted as a guide. Students have come to expect our Number Talks and are prepared for the routine. I've even seen a few of my students playing Number Talks between classes, and been told that they request them in my absence.

A Number Talk routine is minimally invasive to class time, and requires virtually no materials. The gestures are easy for students to recall and for teachers to read. Daily Number Talks get the class practicing many of the abstract habits that leading thinkers in education prescribe. Nebulous as the ideal practice standards can seem at first glance, Number Talks make them quite easy to achieve.

REFERENCES & RESOURCES

Books:

- Hawkins, K. 2014. *The Idea of Arbor School*. Tualatin, Oregon: Arbor Center for Teaching.
- Humphreys, C. & Parker, R. 2015. *Making Number Talks Matter*. Portland, Maine: Stenhouse.
- Sawyer, W. W. 1961. *A Mathematician's Delight*. London: Penguin.

Websites:

- mathforlove.com/lesson/number-races - another version of Number Talks protocol and helpful tips from the Math for Love group in Seattle
- mathperspectives.com/num_talks - a "Number Talks Toolkit" from Kathy Richardson at the Math Perspectives Teacher Development Center of Bellingham, Washington, plus a short, helpful article entitled "What Is The Distinction Between a Lesson and a Number Talk?"
- mathsolutions.com/common-core-support/math-talk/ - Math Talk™ is endorsed by Marilyn Burns' professional development organization, Math Solutions, as key to addressing Common Core standards; hear Marilyn deliver a Math Talk at http://mathsolutions.com/freeresources/post-from-marilyns-blog-a-mental-math-lesson/
- mathtalks.net - middle school teacher Fawn Nguyen's simple website with weeks' worth of problems; resource page includes many links, including a google spreadsheet of additional problems
- youcubed.org - the website of Dr. Jo Boaler at Stanford, including a helpful video introduction to Number Talks at https://www.youcubed.org/from-stanford-onlines-how-to-learn-math-for-teachers-and-parents-number-talks/