VOL. 2, NUMBER 2

Cambium

INNOVATIVE CURRICULUM FROM THE ARBOR SCHOOL OF ARTS & SCIENCES

In this issue

ARBOR SCHOOL
OF ARTS & SCIENCES

POINTS OF ENTRY

Differentiation in mathematics has been an area of focus at Arbor School this year. As a small school with a luxuriously low student-teacher ratio, we pride ourselves on a curriculum that strives to meet children's individual needs as learners. In our mixed-age classes, it's a necessity; disparate skill levels come with the territory. Our teachers devote countless hours to tailoring their lessons, making sure there's a hook for students less inclined toward the subject, an extension for those who will zoom through the material, an inviting way in for those who need scaffolding.

One model for differentiation that feels successful is to develop units that offer many points of entry, allowing a whole class to work on the same problems at different levels of sophistication. We have chosen some of our best to present here. We believe they satisfy what we've come to think of as two keys to good differentiation: entry points that offer visual and/or concrete components and extensions that feel natural and pleasurable, never like punishment for quick mastery. Much of Arbor's thematic curriculum is organized to provide plentiful extensions that offer students the chance to show their understanding through authentic application and creative control. Here you will find fourth- and fifth-graders applying their knowledge of the Pythagorean Theorem to a surveying project, while our youngest students develop critical algebraic thinking by creating their own color pattern sequences and solving for landmark numbers within those sequences.

We try to remind ourselves to be realistic: we can't be differentiating every moment, and there is value for students in the practice of patience and in teaching peers—always with the knowledge that they'll soon get new work that's "just right" for them. Junior teacher Peter ffitch likes to think about the arc of a week's work as he portions out his differentiation efforts; Primary teacher Lori Pressman considers her K-1 class's two years in terms of periods of stretching followed by consolidation.

Ideally, we can help our students practice reaching for agency in their own learning, asking, "How can I make this interesting to me?" "What else does this problem make me wonder that I might have the skills to investigate?" "Can I write a similar but more complicated problem of my own?" Don't wait for the next challenge to be spooned out. Invent it for yourself.

PATTERNS IN THE HARBOR
DEEP PRACTICE FOR PRIMARIES

by Lori Pressman

Patterning, counting, graphing, measuring, adding, subtracting, problem-solving—there are so many different facets of mathematics; leading our K-1 Primaries into this vast system of thinking is an exciting and challenging task. It's fun introducing a new class to our math super hero, Zero, and to new concepts such as odd numbers or AB pattern notation. And it's thrilling to see a child's eyes widen when he realizes ten is written "10" because there is 1 set of ten and 0 units.

Often the biggest challenge in teaching math is providing students with tasks that sufficiently meet their needs, regardless of age or inclination. Each student comes to our K-1 classroom with unique exposure to, interest in, and ideas about mathematics, and designing activities that allow everyone to feel successful is paramount. Over time, the Primary team's problem design has evolved toward differentiation that permits all students to work on the same problem, applying a variety of skills and levels of understanding. Everyone has a point of entry and is able to communicate his thoughts using increasingly sophisticated strategies.

Framing Deep Practice

See Daniel T. Willingham, *Why Don't Students Like School?*

Cognitive research shows that even though children have an enormous capacity for transferring knowledge into long-term memory, their working memories are easily overwhelmed. As novice learners, they can be easily confused by new surface information; recognition that a problem shares a structure with others they have solved can be elusive and requires long practice. In support of that practice, we have constructed a few basic problem frames that recur within a thematic shell connected to our studies of the moment. One such frame involves the children spending a given amount of money to purchase items of varying cost. For instance, during our Journeys curriculum, we draw inspiration from *My Father's Dragon* and visit the Adventure Store to stock up on rope, boots, and tangerines. The students are asked to find multiple item combinations to total the given sum, and many extend the activity on their own to find *all* of the addend possibilities.

Manipulatives are prepared ahead of time and placed on tables so that the children have easy access to concrete models. When preparing for the Adventure Store problem, we used Unifix cubes to represent the different items for sale: 2 brown cubes = $2.00 rope, 4 black cubes = $4.00 black boots, and single orange cubes = $1.00 tangerines. These models are particularly helpful for emergent mathematicians, who can confirm their answers by counting each Unifix cube. Students who need more challenge can be assigned larger sums of money or can add their own items to the store.

Another familiar problem-solving frame focuses on patterns. We often spend time exploring repeating patterns in the fall; our newest mathematicians can practice identifying and building patterns while more experienced peers apply what they know about patterns to make predictions. A favorite activity gives them the pattern and asks them to solve for particular numbers. During our study of plants, the children learned of a farmer who planted a garden row in the following manner: carrot, radish, carrot, radish. Students worked to discover the identity of the 10th, 25th, and 100th vegetables. Familiarity with this problem frame helps students approach the problem with greater confidence and ease: because they have already discovered an entry point for such a problem, they don't waste any time wondering where and how to begin. Instead they can focus on solving the problem and expressing their thinking. They might even find they are ready to try a new strategy suggested by a classmate.

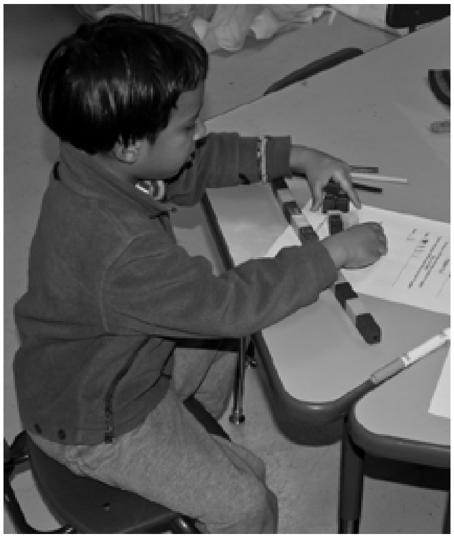
Strategy and Development

Strategy and communication are central to the teaching and learning of mathematics. When we first introduce a problem to the children, we often model a variety of different strategies for solving and recording our findings. Then we set them to work and observe.

What are the first steps taken? Who reaches for the manipulatives and who grabs the markers? Which children are content with a pencil and paper? We're also looking to see who benefits from a reminder about clear communication and who might need some assistance writing numbers.

Strategy and communication are the focus of our assessment of a child's understanding of the problem; instead of rewarding one approach over another, we marvel at the number of different ways to solve the problem and praise clear representation of ideas.

Our young mathematicians seem to follow a common course of development in the communication of their findings—concrete, pictorial, abstract. Beginning problem-solvers often collect the manipulatives they need and then transfer the images of these cubes directly onto their answer sheet. As their understanding develops, however, students start drawing pictures of the items they are representing; now a single image equals x. Some children prefer to show their understanding by writing out the answers, e.g. $4.00 black boots + $2.00 rope = $6.00. All of these modes of communication are appropriate and students can all feel successful regardless of how they have chosen to express their understanding of a problem.



Vivek lines up Unifix cubes to represent the AABB pattern he has designed. Below, Lori helps Nadia count the pink and gray boat shapes she has drawn.

Greg Neps, who teaches math to our middle-grade students, has observed that some students never leave the comfort of visual representations when they are learning a new concept, and he feels that's perfectly acceptable. Any problem that has application (and all K-8 math fits this category) has a concrete representation, and this provides a point of entry for many.

Differentiation

Keeping the developmental course in mind—concrete, pictorial, abstract—we are able to differentiate to meet the needs of the individuals in our class. We can help make a problem more accessible to child who is unsure of how to begin by reaching for the manipulatives or drawing an empty frame of Unifix cubes totaling the sum that we are aiming for. Then the child simply has to find the right number of cube sets and color in the frame. To nudge a child into a deeper level of thinking, we often ask him to record his thinking in a new manner. We might ask the child who has drawn pictures showing her answers to include numbers next to the images or to write a corresponding equation. Substituting larger numbers or introducing the child to a new problem-solving tool are other ways to provide challenge to our hungriest math students. For instance, we might provide a hundreds' chart to help solve the carrot/radish problem. Or we might change the pattern to carrot, radish, corn, bean, carrot, radish, corn, bean.

Conclude with Sharing

Sharing results and strategies with one another is a vital component. Not only does such discussion validate everyone's effort, it also introduces students to new ways of communicating their thinking. We often find students trying new strategies for the next problem-solve, thus developing a more diverse set of tools and a deeper understanding of number and pattern.

Grace has marked all the "black number boats" on a hundreds' chart and is recording those numbers in a column.

The following are examples of problems that we give Primaries over the course of our two years with them. The patterning problems are a series that the children encounter during our study of Journeys; the sets of addend problems are examples from our year of Seasons & Cycles theme.

Patterning Problems

The boats in Mathland Marina are lined up in a pattern—red, yellow, red, yellow.
What is the color of the 10th boat?
What is the color of the 25th boat?
What is the color of the 82nd boat?
Explain how you solved this problem.

In Pattern Port boats are lined up in a pattern—red, yellow, blue, red, yellow, blue.
What is the color of the 10th boat?
What is the color of the 25th boat?
What is the color of the 82nd boat?
Explain how you solved this problem.

You discover a new harbor where the boats are lined up in a pattern. What is the pattern?
What is the color of boat number ___?
What is the color of boat number ___?
What is the color of boat number ___?
Explain how you solved this problem.

Adjustments:
For children who are overwhelmed by these big numbers, choose smaller numbers. We might suggest they find the number of their age or the ages of family members. Often children will draw colored dots or lines representing the boats, but they will lose count. We step in and help them record landmark numbers. For students who are ready for another challenge, we provide laminated hundreds' charts and colored wet-erase pens and have them look for emerging patterns. We also give them more complicated patterns and ask them to solve for the given numbers.

Sets of Addends and a Given Sum

There is about 1 quart of blood in babies, 3 quarts of blood in children, and 5 quarts of blood in adults. Who could be present if there were 15 quarts of blood in a room?

Adjustments:
The total number of quarts in the room could be adjusted higher or lower to accommodate the different needs of the students. Also, you could ask the children to figure out how many quarts of blood are in their family or in the class.

Our class is hatching chicks and butterflies. On Monday morning we discovered 18 legs. What could have hatched?

Adjustments:
The total number of legs could be adjusted either higher or lower to accommodate the different needs of the students. You could also introduce children to a table showing them a systematic means for finding the all the possibilities.

Wenwen said, "I never tried using the hundreds' chart before, but I think I see how to color my boats right on here." She proceeded to color the numbers in her chosen pattern—red, yellow, purple, purple—and then counted along the chart to find the numbers she'd chosen.

A TAPESTRY OF ONE MILLION DOTS

JUNIORS CONSTRUCT LARGE NUMBERS

by Peter ffitch

Dinosaurs last roamed what is now North America some 65 million years ago. The last ice age retreated from these parts just 10,000 years ago, and we know that Native Americans made their homes along the banks of the river that is just a stone's throw from our campus during that same period and remained until just 150 years ago. How can a child who is excited to be turning eight on his next birthday grasp the immensity of these measurements of time? And how can we as teachers support his understanding of the relative value of these large numbers? Those are the questions that we sought to address as we began this year in the Juniors (two blended second- and third-grade classes).

Under the thematic umbrella of "Communities," we spend this year of our two-year cycle learning about the people of North America, what holds them together as groups, what motivates them to move, and how the place in which they live influences their culture. To set the scene, we open the year looking at a prehistoric map of the continental pieces that will become North America. We learn about the dinosaurs that lived where we now live, about the amazing prehistoric mammals that followed them, and then about the first humans to find their way to these shores. We follow these First Peoples as they gradually populate the entire continent, and we pay particular attention to influences of geography, climate, and available resources. As Europeans enter the picture, we keep the focus on the reasons for community and on the "pushes" and "pulls" that moved these groups across the country.

Beyond the Time Line

In past years, time lines have proved useful in helping children to grasp the linear nature of the human story in North America, offering a spatial model for the relative value of large numbers. However, it has been clear to us that the time line model has not given the children a true sense of the immensity of this span of history. The Junior team spent time this summer looking for a way to get a real sense of what 65 million years means relative to the life span of a human being, or even of the human species. Fortunately for us, Mark Girod, Associate Professor of Teacher Education at Western Oregon University and friend of Arbor School, had been thinking about this, too. In his article "Sublime Science," published in the February 2007 issue of Science and Children, Girod presented a lesson that he had designed to help a group of fourth-graders better understand geologic time. With some modifications in consideration of the needs of our younger students, we adopted his lesson and began the creation of the Tapestry of One Million Dots.

We began by giving each child an 8½ x 11 sheet of paper with 10,000 dots printed on it in a 100 x 100 array. With papers in hand, the children moved to the first step without even being asked: how many dots were on each page? Guesses and estimates flowed, and we recorded these numbers on the white board as the children offered them. We did ask that the children make their initial guess at the number without doing any counting, and we found an opportunity for assessment in this process. Some of our younger mathematicians saw what they knew to be a very large number of dots and so guessed what they knew to be a very large number, such as 750, or even 2,000! Others with more large-number practice and a more developed sense of number saw the potential for using multiplication, given a rectangular array. The simple act of writing six- and seven-digit numbers provided an opportunity for place-value work as well.

The dots are small; we played with font size and margins to maximize their size. Our concerns that the dots might be too small and too close together for the children to count were proven unwarranted; the students' younger eyes served them well.

Landmarks to Ten Thousand

With their guesses recorded, we asked the children to gather some information that would help them turn guesses into estimates. Working along the first row of dots, children began by circling the number of dots that equaled their age, and then the number of dots that equaled their teacher's age (I'd conveniently just turned 50), which provided them with a useful landmark number. To confirm that the top row really had 100 dots, some students counted in groups of five, some circled tens, and some felt comfortable eyeballing that the first 50 had gotten them halfway across and that doubling was appropriate. Before continuing, we offered the opportunity for children to make more informed estimates: most refined their guesses considerably.

Now that they were armed with the knowledge that each row contained 100 dots, we asked students to circle 1,000 dots. Those who were ready jumped to the calculation "100 x 100 = 10,000," but even some of those students lacked absolute confidence in the number of zeros they were using. Those who simply counted by 100's inadvertently helped us think about that problem. Reaching "900," our counter named the next number "ten hundred." Calling 1,000 "ten hundred" makes clear to young mathematicians what the value of the number really is, and counting by 1,000's to 10,000 in the same manner made it clear to all of our students that we needed four zeros.

We wrapped up this first day of our big number exploration by taking a moment to use our 10,000 dot papers to help us think about our own ages relative to the number of years since the last ice age and to other spans of time that the children found interesting, such as the life of the world's oldest living person.

Peter and Adlai assemble their strip of 100,000 dots

Building Larger Multiples

We began the second part of this lesson with two goals in mind. First, we wanted children to experience building larger numbers in multiples of 10, having worked from 100 through 1,000 and then to 10,000 on the previous day. We also wanted them to see the immensity of 1,000,000 in relation to 10,000. To accomplish this we had the children work together to construct a quilt of their individual 10,000 dot pages. We gave each pair of children additional sheets so that they had a total of ten. They worked together to tape these papers into a long strip. Counting and calculating as they added each page, students were able to experience for themselves that ten groups of 10,000 is equal to 100,000. All twenty children then came together and, strip by strip, created a quilt of 1 million dots. Before we teachers acknowledged this fact, we gave time for one more round of estimation. How many dots could there be in this nearly 10-foot square quilt? We counted by 10,000's and by 100,000's and even made a T-chart so that students could make a prediction: "If one row = 100,000 and two rows = 200,000…"

With confirmation that we had created a quilt of one million dots, the discussion began again, without any prompting from the teachers, as the children marveled at the size of one million. "Look how small 10,000 years is compared to 1,000,000!" "If this is 1,000,000, imagine how big 65 million would be!" "If we put together 65 million, would it fit on the soccer field?" Children took delight in finding where on the quilt they had circled their own eight or nine years, a number so significant to them but so insignificant relative to the whole. While we acknowledge that it is a challenge for any of us to truly understand the immensity of the numbers that describe the history of the Earth, this lesson brought these children closer to that understanding and, more importantly, filled them with wonder at the scope of it all.

The quilt remains on display in our classroom, a ready reference as big numbers find their way into our daily work. This reference can be made more accessible if increments are marked at 10, 100, 1,000, and so on. With some labeling, the quilt could also serve as a sort of time line, too. If the last dot is marked as the present year, students could count back and mark significant events that occurred within the last one million years.

In subsequent lessons, our mathematicians benefited from stepping back from these large numbers and working with number lines from 0-10 and 0-100. Having had this experience with a visual representation of the relative size of these numbers, a return to the number line emphasized the relative value of the numbers and the spatial relationships that define them.

Download the dot array sheet here: http://www.arborschool.org/pdfs/dotarray.pdf

Junior Down students admire their completed Tapestry of One Million Dots

TRACKING PYTHAGORAS

INTERMEDIATES EXPLORE AND APPLY $A^2+B^2=C^2$

by Daniel Shaw

$A^2 + B^2 = C^2$: The Pythagorean Theorem startles with its simplicity. Three sides of a right triangle related in a single equation. Even though fourth- and fifth-graders (Intermediates at Arbor) possess the skills necessary to apply the theorem with relative ease, a deeper understanding of the concept contributes to a firmer foundation for geometric work to come. In the fourth and fifth grade the ability to view mathematical concepts completely abstractly, without relying on any concrete representation, begins to appear in many students. Students start seeing the concepts themselves: they can add without counting on their fingers, they can do a word problem about fractions without drawing pictures, they think about problems without requiring base-10 blocks. This is not to say that students need to or should avoid concrete representations, but the ability to do math entirely in the abstract is a boon in algebra and advanced math subjects. The study of the Pythagorean Theorem bridges levels of mathematical skill: it can be comfortable for students who need a concrete experience, but also allows challenge and satisfaction for those who can connect with the ideas at an abstract level.

Forecast

This project, a demonstration of the Pythagorean Theorem, aims to solidify students' understanding in some areas as they gain experience in others. Our students had developed strategies for finding the area of a quadrilateral; these proved very useful.

In order to ensure that students understand the concept of area, have them write their own area word problems and exchange with a classmate.

We reviewed some basic concepts about triangles, specifically defining *right triangle*: as one of my students concisely put it, "A triangle with a right angle in it." Our goal was for students to be able to derive and then apply the Pythagorean Theorem to find the unknown side of a right triangle, working first on solving for C (as the theorem states) and then eventually solving for A or B. The final demonstration of their understanding would be in applying the theorem in "real life" using some surveying data students collected on campus.

Different students get to different places along this process. For some, the realization that there exists a relationship between the squares built off of a right triangle presents a sufficient and intriguing challenge. For others, the application to the real world problem is exciting as well as challenging.

Triangles and Squares

To begin the project, we asked students to talk about some of the things they know about triangles and then told them that we were going to experiment with a puzzle to discover some new facts. We asked the students to draw a right triangle on graph paper. The triangle should have legs that create a simple Pythagorean triple. In order to make the math more interesting later in the problem, the triple should have some larger number parts, e.g. (8, 15, 17) or (5, 12, 13). Our class used a triangle with legs 12 and 9 graph paper squares long. The students then drew squares based on each of the legs: one 9 by 9 square and one 12 by 12. Students then cut out the three shapes and set them aside.

Pythagorean triples are sets of three integers that conform to the Pythagorean Theorem: $A^2 + B^2 = C^2$. One example of such a set is 3, 4, and 5.

The primary challenge in this project occurs here. We asked students to imagine a square built upon the hypotenuse (longest side) of the triangle. Then we presented two puzzles. The first is fairly straightforward: What is the length of the long side of the triangle? Students could begin by laying out the 9 x 9 and 12 x 12 squares along the hypotenuse and seeing 6 units extending beyond. Some students also cut out a make-shift graph paper ruler to lay along the edge.

The second puzzle requires a bit more thought: Can you cut up the two squares you have to build the square on the long side? Do you need any extra graph paper? Will you have any left over? The conflict in the second puzzle focused the students' attention and they set right to the task.

Check in with students here. Some will need help "seeing" the square that builds off of the hypotenuse. Finding the length of the long side will help. Ask students how they plan on obtaining that length. Most students should quickly figure out that the length of the long side happens to measure 15 squares.

After solving the first puzzle, students had myriad ways to attack the second. Some cut up the 9 by 9 and 12 by 12 squares into smaller chunks and attempted to put them back together into a 15 by 15 square. Others started by drawing and cutting out a new 15 by 15 square to work with as a base. We encouraged the use of different methods and the sharing of individual strategies as soon as a few students had finished.

Make sure that students prove that the 15 by 15 square is actually composed of the two smaller squares. Expect to see some inefficient strategies, such as cutting out every individual square of the graph paper, and be ready to prompt the students who get stuck by suggesting they calculate the area of each of the squares.

Whatever the strategy, all our Intermediates eventually found that they had exactly enough graph paper squares to build a 15 by 15 square.

Writing Toward the Theorem

The next step brings writing into the math class and helps students clarify their own reasoning. Writing out their thinking assists students in working toward their own demonstration of the Pythagorean Theorem. We asked students to begin by describing exactly how they found out that the 15 by 15 square could be made with the smaller two squares. Fourth-grader Lily worked calculations and wrote, "I found the area by x 12 and 12, and then 9 and 9. Then I added the answers of those. 12 x 12 = 144 + 9 x 9 = 81, 81 + 144 = 225." In calculating the large square's area, she added, "This time I multiplied 15 x 15. I got the same answer as last time, so I'm guessing that it's right. 15 x 15 = 225." Her classmate Joe chose a more concrete approach to reach the same conclusion. He wrote, "I took the 9 square and cut it into lots of 3 squares. I put those up against the 12 square and that made it a 15 square, and that was the length of the triangle. I used all of the 9 square and all of the 12 square. That means that 15 x 15 = 12 x 12 + 9 x 9." Meanwhile, Lewis made the leap to the theorem itself and wrote that, in a right triangle, $A^2 + B^2 = C^2$.

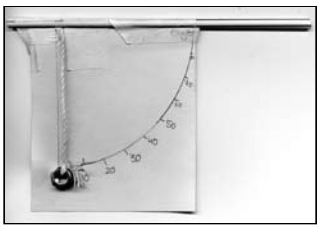
Many students will only get to this point in the course of three or four lessons. This is a great point for them to stop if they are not yet ready for the abstraction of the theorem. Ensure that they understand that the area of the 15 by 15 square equals the sum of the areas of the smaller two squares. Their writing will make their understanding plain.

We stressed that this should be an exercise in effective writing as well as math. We expect full sentences, clear handwriting, and logically ordered paragraphs at this grade level. However, Beatrice's meticulous instructions for solving the problem eschew paragraph form, substituting a lift-the-flap model and a scale diagram to show the most effective subdivision and redistribution of the 9 x 9 square to form the 15 x 15 square. Her thought process and supporting explanations were clear. This assessment offers an intriguing view of each student's thinking and command of mathematical strategies.

After students have written about how they discovered the fact that the area of the square on the hypotenuse equals the sum of the areas of the smaller two squares, they are ready to try to write the Pythagorean Theorem itself. We asked our students to make some guesses, suggesting some right triangles with Pythagorean triples as legs to experiment with. Is the area of the square on the hypotenuse always equal to the sum of the areas of the squares on the legs? How can you write that fact in words? Many students can write the theorem in words, something like this: "In a right triangle the area of the two small squares adds up to the area of the big one." Some have the ability to express this sentence symbolically and arrive independently at $A^2 + B^2 = C^2$. Armed with the theorem itself, students can start reasoning about the how the theorem can be used. What if we didn't know one side of a right triangle? We presented our students with simple triangles, where all three legs are integers and the hypotenuse is the missing side.

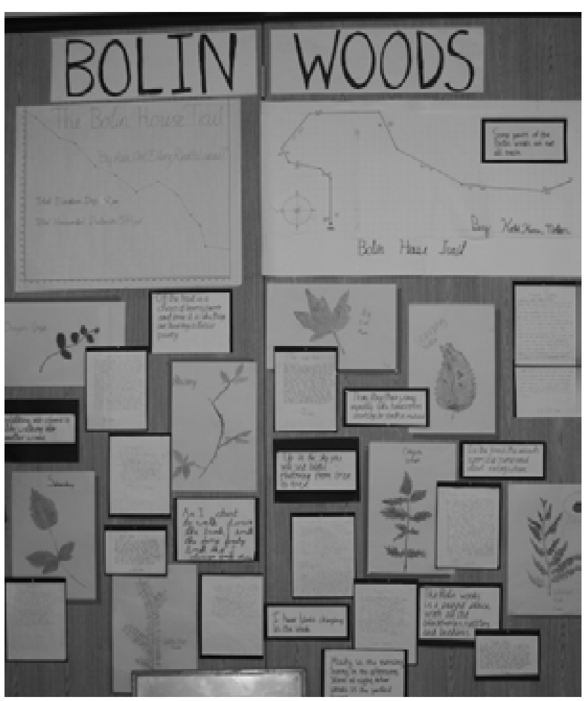
Working in groups at this stage was helpful to students as they figured out how to apply the theorem. Below, Lily and Olivia compare strategies.

Advanced mathematicians will be able to tackle problems with non-integer values by estimating square roots. We gave these sorts of problems to any who were willing to attempt them.



A homemade surveyor's scope

Students can easily help craft these basic surveying tools. Have students cut a 10' section of PVC pipe and mark every half foot with colored electrical tape. This pole will let the student looking through the scope measure her own change in elevation. To make a scope, students can attach a straw to a note card. A metal nut tied to a string acts as a plumb line to ensure the scope is level and the reading accurate. (A pocket or keychain level would work, too.) After the construction of our tools, every student could participate in this work, independent of his level of understanding of the Pythagorean Theorem.



Application

Our final application (for this unit, at least) of the Pythagorean Theorem occurred outside as part of the survey work the Intermediates are completing in order to design and, ultimately, build a cross country trail around Arbor's campus. Students used a simple scope-and-pole technique to measure the elevation change on hilly segments of the trail. Small teams calculated the elevation change with the scope (representing side B of a right triangle) and the slope of the trail with a tape measure (side C of a right triangle). Back in the classroom, they prepared to create an elevation map of the trail section we had measured. In order to do so, they needed to find the missing side of the right triangle (side A), the distance between the scope and the elevation pole. Working with numbers that they pulled from the real world capped the work with the Pythagorean Theorem nicely, and the Intermediates created beautiful displays with their elevation maps, poetry, botanical illustrations, and descriptions of the plant species found near each segment of the trail they had surveyed. Students came out of the Pythagorean work feeling that they had discovered a formula that is not only new to them, but useful as well.

THE CROW AND THE PITCHER
SENIORS EXPERIMENT WITH TWO VARIABLES

by Greg Neps

This lesson, typically done by Arbor students during the summer break between their sixth- and seventh-grade years, is an experimental introduction to two-variable equations. Our rising Sevens have had much practice working with, solving, and graphing single-variable equations in the math classroom and have had a year of science lab work to support graphing two-variable experiments. This assignment explicitly brings the two together. While some students toil over this experiment for days with exacting care, others quite quickly make the connections between the experiment and the underlying algebra of their formula. In the classroom, the experimental portion might take some students a single class period while others could easily spend a week in exploration.

In approaching this work without the guidance of a teacher, students end up applying a wide array of skills and concepts. Some, with sibling or parental support, may have explored the Cartesian plane, slope ratios, or slope-intercept form, while others will have simply done the experiment and graphed their data points.

The seventh-grade curriculum at Arbor explores the interplay between variables and how one can express math relationships both graphically and algebraically—seeing the geometry behind the algebra. This experiment provides students with a common project, language, and experience from which they can draw as they extend their understanding through in-class discussions and activities.

This experiment could be easily adapted for any classroom fifth-grade through eighth-grade, depending on the concepts your group is studying.

Experiment and Data Representation

Working individually or in pairs, the students will set up and conduct the following experiment. Samples of student answers are included.

Materials for each student:

- Marbles or glass beads, uniform in size, approximately 80-100
- Tall, narrow, transparent container—a tennis-ball tube works well
- Metric ruler
- Water
- Notebook and pencil

According to Aesop:

A crow perishing with thirst saw a pitcher and, hoping to find water, flew to it with delight. When he reached it, he discovered to his grief that it contained so little water that he could not possibly get at it. He tried everything he could think of to reach the water, but all his efforts were in vain. At last he collected as many stones as he could carry and dropped them one by one with his beak into the pitcher, until he brought the water within his reach and thus saved his life.

Today you get to be the crow—clever, resourceful, patient, and hardworking. You also get to have some fun exploring just how long it may have taken Aesop's crow to fill that pitcher. In fact, by the end of this experiment you will be able to predict how many stones the crow would need regardless of the height of the container.

Procedure

1. Fill the container about halfway with water, ensuring that, when you hold the ruler to it, the water height is a whole number in centimeters.
2. Record the height in the first line of your data table (*see next page*).
3. Carefully add marbles, one at a time, until the water level has risen 1 cm.
4. Record this number of marbles on your data table.
5. Accurately measure the new height of the water in the cylinder and record this in your data table.
6. Continue adding marbles, pausing each time you have raised the water level an additional 1 cm to record the number of marbles used and the new water height.

Number of marbles (x)	0									
Height of water in cm (y)										

The two variables in your experiment are x (the number of marbles) and y (the height of the water). Which variable do you think is the dependent variable? Why?

"I think that the water level is the dependent variable because it depends on the marbles to fluctuate." –Theo

Which one is the independent variable? Justify your answer.

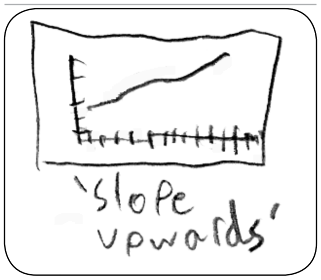
"I think that x is the independent variable because it causes the y variable to change and doesn't get changed by y." –Dylan

When two variables are combined visually in a graph, the horizontal edge of the graph is called the "x-axis." What information should be displayed along the x-axis?

"Number of marbles, because the water level/height should actually be height-ish, like... going up." –Theo

What do you think the vertical edge of your graph is called? How will you label it?

"The y-axis, and you would put the height of the water on the y-axis." –Connor



Theo's graph prediction sketch

Before you turn your data into a graph, predict the shape you think it will take. Why would you predict that shape?

"It will slope upwards, because the water level is getting steadily higher with every marble added." –Theo

"An upwards diagonal line, because when x grows, y grows." –Connor

Now that you have collected all that information, it's time to represent your data so that we can easily compare our findings. The best way to do that is to present the data in two uniform ways: graphically and algebraically. Graphically should be pretty simple, since you have all the data points and labels already figured out. Plot the points, letting 1 cm equal 1 unit on the vertical axis. You can decide how many marbles should equal 1 unit on the x-axis.

If you connect your data points, what type of graph is it? *"It's a line graph."* –Olivia

Where does the line that you made cross the y-axis? Or, in simpler terms, what was the height of the water before adding marbles? *Answers will vary.*

Many students did not have sufficient marbles of identical size to perform the experiment. Connor and Olivia both graphed the actual data produced by their motley collections, then plotted a line of best fit to represent their predictions for an experiment using equal-size marbles.

As a common or simplified fraction, what is the ratio of height increase to the number of marbles placed into the container? Or in simpler terms, what is the rate of change in terms of cm/marbles?

Answers will vary. "Mine varied from 1/9 to 1/13 because the size of my marbles was different." –Connor

Interpolation, Extrapolation, and the Limitations

In small groups, the students will explore each other's graphs and puzzle out the introductory algebra of linear formulae. The groups should consider these questions:

- By examining the graph, how can you tell the water height at the beginning of someone else's experiment?

• By examining the data table, how can you tell the water height at the beginning of someone else's experiment?

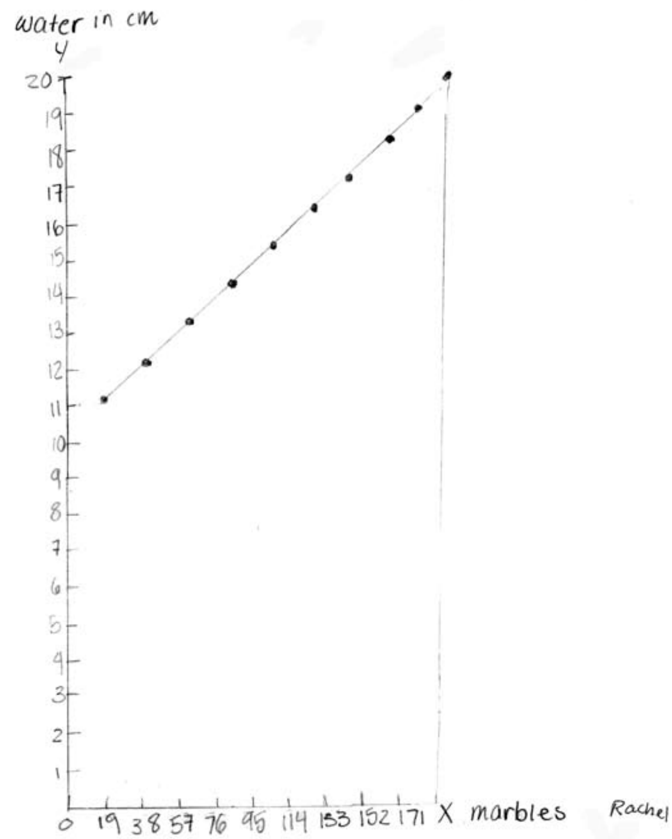
• How do the data table's starting point and the graph's starting point relate?

• How many marbles would it take to raise their water level 1 cm?

• How many marbles would it take to raise their water level 2.5 cm? How did you calculate this?

• Examine the graphs at your table and discuss with your group why some of your lines slope upward more rapidly than other people's graphs. Talk to each other about experimental design, equipment, and procedures to see if you can agree on an explanation.

Number of marbles (x)	0	19	38	57	76	95	114	133	152	171
Height of water in cm (y)	11 cm	12 cm	13 cm	14 cm	15 cm	16 cm	17 cm	18 cm	19 cm	20 cm



Rachel's graph shows a steep slope because she let each new value of x represent one unit on the x-axis. Other Sevens chose to plot 2 marbles or 5 marbles as 1 unit. Make sure students notice and adjust for these differences during their group work.

• Name two things that can affect the rate at which the water level rises (other than the number of marbles). How do those two things (variables) impact your experiments and your graphs?

• If your "pitcher" were 3 meters tall, how many marbles would you need to get the water to the top? Describe how you made this estimate.

• What if the pitcher were 30 meters tall?

Your answers for the last two questions give us estimates as to how many marbles we would need, but your experiment gives us data we can turn into a formula to determine exactly how many marbles would be needed.

From your data and the questions you have answered, create an equation that would allow you to predict information not found on your graph. (Hints: What do you have to do to x to get y? What value does y have when x is zero?)

y = ___________________________

Often equations of lines are in this form:

y = (the rate of change)x + the initial starting point

Check your estimates using the formula for your experiment. How different are your answers?

Use your formula to determine the number of marbles needed to get the water to the top of a pitcher 8,848 m high (the height of Mount Everest).

"The Crow and the Pitcher" is, effectively, Chapter 0 of our seventh-grade textbook. Some of our working groups were able to race through these extrapolation questions and move on to Chapter 1 material, while others needed more time and support. For younger or less experienced students, we select just a few of the interpolation and extrapolation questions and build in more time for whole-group strategizing about possible approaches.

If each marble weighs 12 grams, how many metric tons of marbles would you need? (1,000 kg/metric ton)

Clearly the last couple of questions have some flaws (as does the formula you created); what happens as you continue to place marbles into a container with a limited amount of water?

Does the water level always continue to rise as you add marbles? Describe the flaw in this experiment and describe why the graph will cease being linear.

Sketch a graph of how this experiment might look if we continued to add marbles.

Extensions

Create, describe, and graph an imaginary situation in which one variable depends on another and explain why the graph is shaped the way it is. Example: Greg's hairline seems to have changed as he has gotten older… x = Greg's age, y = the distance from his hairline to his eyebrows.

"Big E everyday eats twice the amount of candy she ate yesterday. For this graph, x = days and y = pieces of candy." –Olivia

"If Mo gets paid $5 every week to mow Mr. MacPherguson's lawn, how much money would he have after 2 months?" –Ben

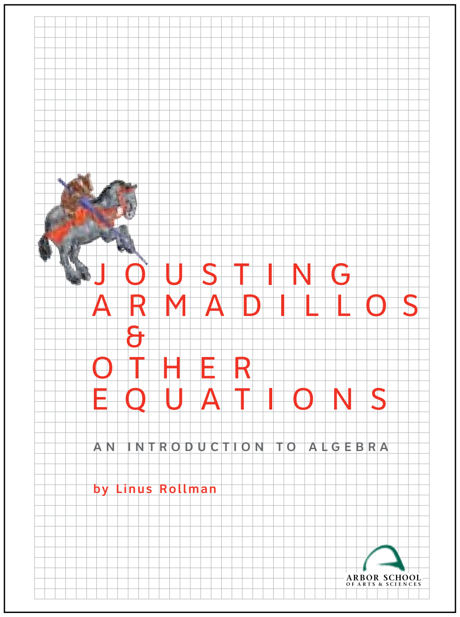
Dylan graphed her puppy's growth in inches per month; Rachel graphed the annual increase in her paper clip collection.

Checking in with students about their understanding of dependent and independent variables is important; you will probably see some dependent variables plotted on the y-axis.

How might you create an experiment that has no slope (a horizontal line)? What could be variables in this experiment?

Further Application

We use the Crow and the Pitcher work as a springboard and a regular reference for in-class work with the Sevens. They have taken ownership of their equations and can therefore more clearly see how they might apply the algebra of two-variable equations, with which they will become intimately familiar in the course of their seventh-grade year. Not once during the opening months of the school year have we heard, "When am I ever going to use this stuff?" However they come into their new classroom, it is possible to meet students where they are and meaningfully engage them in the classroom activities on many levels.



Jousting Armadillos & Other Equations: An Introduction to Algebra is a new textbook by Arbor teacher Linus Rollman. This text, the first in a trilogy of Arbor Algebra books, presents our sixth-grade math curriculum. Its aim is to invite and engage students in beginning algebra, allowing them to find success and pleasure in this new realm of mathematics.

From the Introduction:

[*Jousting Armadillos*] is structured for the sort of independent, small-group work that happens in Arbor math classrooms. The book probably cannot actually be experienced by a sixth-grade student entirely without teacher mediation and assistance. But that level of independence was the goal I was aiming for, knowing that I would fail—the line that I was trying to approach asymptotically, if you like. It's not a book that is meant to be supplemented by many lectures or by a teacher working through example problems on an overhead projector. It's written for kids, which means that the tone is a little different from many textbooks (though not, I fervently hope, patronizing) and it's written to be discussed and debated—by students and their classmates, by students and their families, by students and their teachers—rather than taught.

…

[Another] thing that sets this book apart is its emphasis on writing. It contains many "problems" that ask the kids to write. Sometimes they're asked to brainstorm lists, sometimes to record their understandings, sometimes to create puzzles for one another, sometimes to free-associate. There is a wealth of literature on tying writing into areas of teaching that don't traditionally involve a great deal of writing. I suppose it's clear that I believe there's real value to that idea. (That may not be surprising; my own academic background is in the humanities rather than in math.) I'll say up front that if you're not willing to give some credence to that basic notion, this is probably not the textbook for you to use in your classroom. I'll also say that, while Arbor's math program has its weaknesses, as all do, one area in which we've undeniably done well is in holding the interest of students who, in another setting, might have considered themselves "non-math" people. I think the integration of writing into the program is one powerful reason for that.

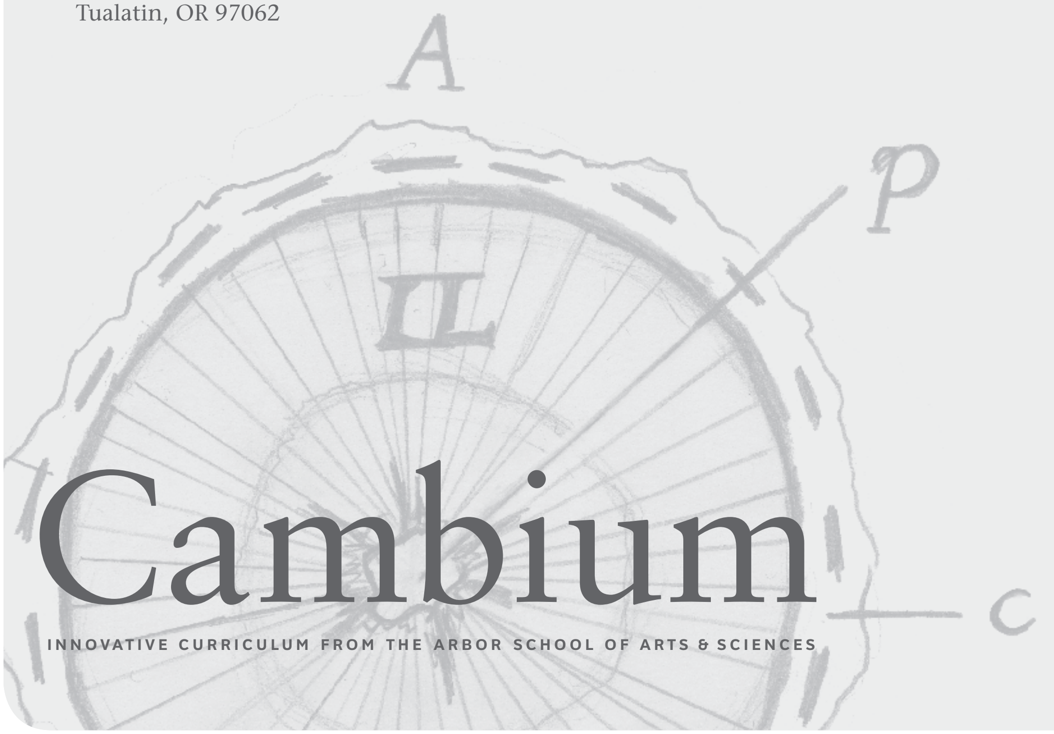
…

I began by saying that we used to work from several textbooks and that they weren't, even in combination, quite perfect for our purposes. No textbook can be perfect for everyone's purposes and this one certainly won't be. (I'm especially curious about how this book might fare in a classroom with a significantly higher student/teacher ratio.) So, please, make modifications. Use the sections that you find interesting or useful and discard the others. Add things. Change things. And please, give us your suggestions, whether you're a teacher, a parent, or a student. The theory has always been that this will be a "live" text—one with many authors that is constantly being re-shaped.... I would love to hear from you. *–Linus*

If you are interested in receiving a copy of *Jousting Armadillos* for review, please contact Sarah Pope at icci@arborschool.org, or by telephone at 503.638.6399.

The Arbor class of 2011 helped to shape this text as they worked from the draft during the 2008-09 school year. The Answer Book is largely their own work. The same students are now applying themselves to the second installment in the trilogy, while our Eights are formulating some of the problem sets as they review and build upon last year's knowledge.

ARBOR SCHOOL
OF ARTS & SCIENCES
4201 SW Borland Road
Tualatin, OR 97062



Cambium

INNOVATIVE CURRICULUM FROM THE ARBOR SCHOOL OF ARTS & SCIENCES

THE INTELLECT, CHARACTER, AND CREATIVITY INSTITUTE AT ARBOR SCHOOL OF ARTS & SCIENCES

Arbor Director: Kit Abel Hawkins
ICCI Director: Annmarie Chesebro
Editor: Sarah Pope
Design: Mary Elliott
Photos: Sarah Pope, Peter ffitch

4201 SW Borland Rd.
Tualatin, OR 97062
503.638.6399
cambium@arborschool.org

Cambium: (n) the cellular growth tissue of trees and other woody plants, from medieval Latin "change; exchange."

What content would you like to see offered in Cambium? Do you have ideas to improve it? Please send us an email: cambium@arborschool.org.

Masthead by Jake Grant, after an 1890 botanical illustration. Plant block print by Annika Lovestrand.

The Arbor School of Arts & Sciences *is a non-profit, independent elementary school serving grades K-8 on a 21-acre campus near Portland, OR. Low student-teacher ratios and mixed-age class groupings that keep children with the same teacher for two years support each child as an individual and foster a sense of belonging and community. An Arbor education means active engagement in learning, concrete experiences, and interdisciplinary work.*
For more information on the Arbor philosophy, please visit www.arborschool.org.

ICCI *is a private, non-profit organization created to train teachers in the Arbor educational philosophy through a two-year apprenticeship while they earn MAT degrees and licenses, and to offer guidance to leaders of other independent schools.*
ICCI is now accepting applications for the 2010-2012 cohort of apprentices.

Wenwen spends $6 at the Adventure Store: $4 black boots + 2 $1 tangerines

NEXT ISSUE:
THE ARTS AS CORE CONTENT

Subscribe to Cambium now to make sure the next issue of the 2009-2010 school year arrives in your mailbox this winter! Return the subscription card enclosed with a check for $20 to "ICCI at Arbor School" to receive the four 2009-10 print issues. If you'd like to receive Cambium solely as a PDF file by email, the cost is $10.